Sovereign Bailouts and Senior Loans

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Abstract

Institutional lending in crisis is evaluated from a theoretical point of view. First, the share of senior loans in new loans is irrelevant under a given probability distribution of the country's resources. Second, seniority may partially alleviate the inefficiency of debt contracts when the distribution of resources is endogenous to the

country's physical investment and effort towards success. Third, with multiple lending rate equilibria, institutional lending may induce a switch to a lower private loan rate if it can be done in a sufficiently large amount. Fourth, conditions are analyzed under which debt forgiveness is efficient under a financial shock.

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1 Introduction

There have been three official bailouts of countries in the euro periphery starting in 2010. The bailout of Greece, which was finalized over April-June 2010, was followed by similar bailouts of Ireland (November 2010) and Portugal (May 2011). Financial rescue packages and fiscal programs have been jointly orchestrated by the IMF and Eurogroup, with funding from the latter channeled via the European Financial Stability Facility created in May 2010. A second bailout for Greece was approved in March 2012 following a 70-75 percent haircut in net present value terms on government bonds held by private creditors. These bailouts have had one feature in common: Except fleetingly, none served to quieten the markets and lower bond spreads to levels that would connote low default probabilities as one might have expected given the size of the announced bailout packages and the international muscle behind them. To the contrary, bond spreads rose significantly following the implementation of the bailouts. In other words, the situation worsened.

Several reasons could potentially explain why the debt outlook for these countries worsened in the wake of the initial bailout announcements and the start of their implementation in spite of progressively bigger official sums being put on the table. These include news about hidden deficits and debt, the lack of a credible fiscal adjustment program and political impediments to the needed austerity, or to design flaws in the bailout packages.

In our assessment of Greece (Chamley and Pinto 2011), we argued that the main reason official bailouts of sovereigns tend not to work is the negative interaction between the seniority of the official loan and the insolvency of the government. The immediate purpose of the official loan is to prevent a default on short-term debt by paying it off in full when it matures. The deeper purpose is, via reform, to raise the present value of future primary surpluses sufficiently so that default even on long-term debt is avoided. However, if long-term bondholders are skeptical about the program's ability to raise primary surpluses, they will perceive their claims as devalued relative to short-term creditors, who exit at 100 cents on the dollar facilitated by the arrival of the official loan. This would prompt a selloff and a rise in long-term bond spreads, which is what

we have witnessed in practice. In such a case, an upfront haircut for all creditors would lead to more easily attainable fiscal targets (as less official money would be needed).

What is the impact of official seniority in such a case? Being senior means having the first claim on available primary surpluses remaining after the exit of the short-term creditors. But this is irrelevant to the extent that official loans are priced in line with the risk they bear: the key point is the exit of short-term creditors at 100 cents on the dollar. However, seniority could become an issue if official loans are priced above the risk-free rate in spite of their seniority. In a related paper, Gros (2010) argued that the formal announcement of seniority could cause the sort of problems described in Chamley and Pinto (2011). He attributes the problems with the Irish bailout in part to the November 2010 announcement that official financing from the Eurogroup would be considered senior to private loans; but the case of Greece itself as well as that of Russia in 1998 (Kharas, Pinto and Ulatov 2001) convincingly shows that all that matters is a perception of seniority based on precedents. However, it is important to note that seniority *per se* is unimportant so long as the official loan is priced in line with its risk.

The seniority clause that often included in official loans⁴ has been criticized because it could weaken the incentive of private lenders⁵. We focus on this issue and we argue that there may be some misconception in that debate⁶. One should make a distinction between the impact on new loans that are made in conjunction with the IMF loan, and the impact on existing loans with a service due at the time of the IMF loan. We address the first issue in the next section and provide some discussion of the second in Section 4.

⁴ However, loans by the European Financial Stability Facility (EFSF) that was set in May 2010, as a temporary measure to be replaced by the European Stability Mechanism (ESM), were not senior to private loans. Also, the European Central Bank has recently announced (August 2, 2012) that its purchases of euro area bonds will not transform those bonds in “senior bonds”, as was in the case in the past.

⁵ See also the article A 10 of the ESM.

⁶ A number of studies on seniority (Penalver (2004), Bolton and Jeanne (2005), Conesa and Kehoe (2011)), address issues that are different from the ones in this paper.

A senior official loan obviously increases the interest rate on new (private) loans that are made in the same environment. But the analysis in Section 2 shows that this is either neutral or good for the country. If the loans are made only to fill a liquidity gap and have no impact on the distribution of future resources that will service the loans, the mix between senior and junior debt is irrelevant. The junior debt is more risky and must command a higher rate, but the total debt service is unaffected. When the default depends on the total debt service and some random resources, there is a Modigliani-Miller argument about the seniority of part of the debt (Proposition 1).

When the probability distribution of resources depends on some decisions by the borrowing country, (*e.g.*, physical investment, fiscal efforts to raise the level of resources), then the higher rate on the junior debt improves the welfare of the country (Proposition 2): it alleviates some of the inefficiencies of the debt contract that cannot be written with payments contingent on the decision variables of the borrower. We apply the analysis of this section to the concept of “Blue Bonds” (Delpla and Weizsäcker, 2011).

In Section 3, we turn to the issue of multiple equilibrium lending rates. A high rate may be self-fulfilling because it increases the probability of default. It is shown that a small intervention by an official institution has no impact. Only sufficiently large interventions may trigger a switch to an equilibrium with a lower rate on private loans.

Section 4 considers the case of a country that needs new financing because of an adverse shock. Following the remarks at the beginning of this introduction, should new loans be used to service the old debt or should the old debt share part of the financial shock? In Chamley and Pinto (2011), we showed how the absence of a haircut on the old debt could worsen the fiscal situation of the borrower in the future. We expand on that framework by including the cost of debt forgiveness. At issue is the balance between the immediate cost of debt reduction versus the cost in the future. We analyze this trade-off in a simple model and characterize some conditions for immediate debt reduction.

2 One-period debt contracts and sovereign loans

Sovereign loans are in the form of debt contracts with a repayment schedule. The efficiency of the standard debt contract between two private parties has been demonstrated by Townsend (1979) and Gale and Hellwig (1985) in a one period framework: the fixed schedule saves on the cost of verification and negotiations for all the states in which the borrower can pay. If the borrower cannot pay, then that cost is incurred. Under this event, bankruptcy, it is efficient to transfer as much as possible to the lender so that the lender, expecting the transfer, minimizes the lending rate *ex ante* and therefore the fixed payment that triggers the bankruptcy.

In sovereign loans, the rule about the transfer may not be strictly applicable, but a similar principle is at work: the higher the payment that lenders expect if the country defaults, the smaller the *ex ante* rate and therefore the smaller the probability of default, for a given probability distribution of resources. The cost of monitoring in sovereign lending and coordination between lenders is probably even higher than for private firms (*e.g.*, the missions of international organizations to Greece). That high cost of monitoring may be one of the reasons for the lack of sovereign equity lending.

Institutional lending by the IMF, or other international organizations (*e.g.*, ESM (European Stability Mechanism) loan to Spain), is typically senior to other loans. Even Argentina, after its 2001 crisis, paid back its loan in full including interest to the IMF. It is often argued that such seniority reduces the incentives for private lending and is thus counter-productive for the catalytic effect by which IMF lending should stimulate private lending. In that debate, there may be some confusion between the impacts on new and old loans, respectively. A new loan that is senior to past loans trivially reduces the value of these loans if it does not sufficiently increase the probability of higher resources for the debt servicing. The new loan with seniority may therefore depress the value of old loans, as seems to have occurred, for example, in June/July 2012 for the old Spanish 10 year bonds. In this section, we focus on the impact of senior lending (*e.g.*, by institutions) on the interest rate on new loans that are made at the same time by private investors. We first assume that the probability distribution of the borrowers' resources is exogenous.

2.1 Exogenous resources

To focus on the main argument, consider a country that borrows a fixed amount that is normalized to 1 for one period, between periods 0 and 1. Assume that the level of resources that is available to pay the loan is a random variable y that is realized in period 1 with a probability density $f(y)$. Let b be the amount that is due in period 1, to be determined later. If $y \geq b$, the country meets the payment b . If $y < b$, the country defaults⁷. By assumption, default reduces the amount that can be captured by creditors (either because of monitoring costs or because the country can divert some resources for other purposes), and that cost is a function $\tau(y, b)$.

We assume that private lenders lend at the rate R (principal and interest), in order to get, *ex ante*, the risk-free rate R^* . From the previous assumptions, the private rate satisfies the equation

$$R^* = R \int_R f(y)dy + \int_0^R (y - \tau(y, b))f(y)dy. \quad (1)$$

One should note that this equation may have multiple solutions in R . We will examine this issue later, in Section 3. Assume now that a fraction m of the loan is granted by an official institution (*e.g.*, the IMF), at the rate $\bar{R}$, and the balance is supplied by the market. Following the historical evidence, we assume that the country has always sufficient resources to pay the official loan at the risk-free rate R^* . The contractual payment that the country has to pay to avoid default is now

$$b = \bar{R}m + R(1 - m), \quad (2)$$

and the previous equation for the private rate now becomes

$$R^*(1 - m) = R(1 - m) \int_b f(y)dy + \int_0^b (y - \tau(y, b) - \bar{R}m)f(y)dy. \quad (3)$$

Rearranging the terms and using (2),

$$R^* + m(\bar{R} - R^*) = b \int_b f(y)dy + \int_0^b (y - \tau(y, b))f(y)dy. \quad (4)$$

⁷ One could also consider a situation where the country defaults if the penalty for default is smaller than the benefits from not paying.

This equation has a simple interpretation. The left-hand side measures the expected return on all loans, private and official. Recall that the official loan is risk-free. Hence an official rate $\bar{R}$ above the risk-free rate R^* operates like a tax, and it is a subsidy if $\bar{R} < R^*$. By arbitrage, the left-hand side has to be equal to the right-hand side that measures the expected value of the sum of all the debt payments.

When the official rate is equal to the risk-free rate, $\bar{R} - R^* = 0$, the amount of the official loan, m , has no impact on the debt payment b , and therefore on the probability of default. The senior component of the loan increases the interest rate on the private loans since these loans face a greater risk, but the total value of all loans made at the same time, institutional with seniority and junior private loans, is independent of the composition between the two components. The independence of the default from the composition of the debt is similar to the Modigliani-Miller theorem⁸.

Proposition 1 (Modigliani-Miller)

Assume that in the previous model, (i) there is an equilibrium with a one period debt of 1 in private loans $1 - m$ at the rate R and an institutional loan m at the rate $\bar{R}$; (ii) the risk-free rate R^ is equal to the ex ante expected rate of the private and institutional loans. Then the debt payment (equal to the threshold level of income that triggers default) is independent of the amount of the institutional loan m .*

We assumed for simplicity that $y \geq R^*m$ in order to fit with a risk-free loan and the IMF requirement that it be repaid. But the result holds even when the senior claim is risky (and its rate adjusted to keep the *ex ante* value unchanged). As with any Modigliani-Miller type result, the property is simple, but the property should be emphasized in order to dispel some notions that seniority in official loans weaken the incentive of private lenders.

If the official loan is at a rate that is higher than the risk-free rate, the premium operates as a tax that reduces the level of net available resources in period 1 and it

⁸For the Modigliani-Miller theorem with respect to the composition of the central bank's balance sheet, see Chamley and Polemarchakis (1984).

increases the probability of default. Likewise, a loan at a lower rate operates as a subsidy and reduces the contractual payment that is due.

The property can be restated in different terms: an institution such as the IMF can charge a lower rate because of its senior claim. That lower rate reduces the debt service of the country. The remaining part of the debt is more risky, obviously. But that higher risk is associated with a higher loan rate on a debt that is reduced by the amount of the IMF loan. The total service is not changed, and if the default decision is triggered by the relation between the debt service and some other variable (*e.g.*, future income or penalty for default), then the default decision is not affected by the seniority clause.

2.2 Moral hazard

So far, the probability distribution of resources for loan repayment has been exogenous. However, all institutional loans are made in the context of a crisis where the borrowing country has to make some effort toward increasing resources. To analyze the issue, we consider the following model.

The model

There are two periods. In the first period, the country borrows. In the second period, the country's resources that are available for loan repayment, y , are defined such that

$$y = \begin{cases} a + f(k), & \text{with probability } \lambda(e), \\ a, & \text{with probability } 1 - \lambda(e), \end{cases} \quad (5)$$

where k is the capital invested by the country and e is the level of "effort" made by the country. That effort is not observable or cannot be used in a contingent contract. The function $\lambda(e)$ is such that

$$\lambda'(e) > 0, \quad \lambda''(e) < 0.$$

The parameter a represents a minimum level of resources that is available for loan servicing and k is a level of investment, to be financed by foreign loans. The opportunity cost of foreign lenders is the risk-free rate R^* (including repayment of the principal).

Equilibrium without seniority

Consider first the case where all loans to the country have the same seniority and therefore the same rate R . The country defaults when the realization of the random resources is zero. Assuming competitive lending, the rate R is determined by the arbitrage equation

$$R^* = \lambda(e)R + (1 - \lambda(e))\frac{a}{k}. \quad (6)$$

The country's welfare is an increasing function of the level of resources net of loan repayment and a decreasing function of effort. Assuming an additive structure and normalizing the level of effort, the welfare is set to be equal to

$$U = \lambda(e)(a + f(k) - Rk) - e. \quad (7)$$

The country borrows an amount k and produces a level of effort e such that

$$\begin{cases} f'(k) = R, \\ \lambda'(e)(a + f(k) - Rk) = 1. \end{cases} \quad (8)$$

The first best

Consider a planner who adds the utilities of the borrowing country and of the lenders in the welfare function

$$J = a + \lambda(e)f(k) - e - R^*k. \quad (9)$$

The optimal allocation is determined by the conditions

$$\begin{cases} \lambda(e)f'(k) = R^*, \\ \lambda'(e)f(k) = 1. \end{cases} \quad (10)$$

In order to compare the allocations in the first best and in the equilibrium without seniority, assume that a is negligible. In that case $R^* \approx \lambda R$ in (6). Let k and k^* be the levels of investment in the equilibrium and in the first best, respectively, and e and e^* the levels of effort. The first-order conditions for the investment are the same in the equilibrium (8) and in the first-best (10) with $\lambda(e)f'(k) = R^* = \lambda(e^*)f'(k^*)$. The equations that determine effort are different, however. In the first-best, $\lambda'(e^*) = 1/f(k^*)$, whereas in the equilibrium, $\lambda(e) = 1/(f(k) - Rk)$. The values k^* and k are

different, but one can verify that if the degree of concavity of the probability of success $\lambda(e)$ is sufficiently high, then $e^* > e$. In the equilibrium, effort is smaller than in the first-best.

If $a > 0$, then from (6) in equilibrium, $\lambda(e)R < R^*$ and $\lambda(e)f'(k) < R^*$. Comparing with the first best rule in (10), the marginal productivity of capital is lower in the competitive equilibrium (for a given level of effort). In the competitive equilibrium without seniority, there is too much investment in the sense that the marginal productivity of investment is smaller than the borrowing cost of the country. The cost of borrowing that is too low. The separation of the loans between senior and junior loans will not generate the first-best allocation but it will partially remedy the inefficiency that has been shown here.

Senior loans

We assume now that $a > 0$, and that an institution makes a loan L , such that $R^*L \leq a$, with a seniority clause. Given this inequality, the institution can lend at the risk-free rate R^* . The loan L is taken as exogenous by the borrowing country which borrows $b = k - L$ on the private market, in order to make a total investment of k . As we have seen in the previous section with exogenous resources, the impact of seniority is to raise the interest rate of the private loans. Given the above comparison of the first best and the equilibrium without seniority, we can conjecture that an increase of the borrowing cost may be welfare improving. We now investigate this issue.

From (7), the welfare of the country takes the form

$$U(b, e; R, L) = \lambda(e)(a + f(L + b) - Rb - R^*L) - e, \quad (11)$$

and the lending rate is determined by the no-arbitrage equation

$$R^* = \lambda(e)R + (1 - \lambda(e))\frac{a - R^*L}{b}. \quad (12)$$

From (18), the country chooses a level of borrowing b , and of effort e , to satisfy the first-order conditions

$$\begin{cases} f'(k) = R, \\ \lambda'(e)(a + f(k) - R(k - L) - R^*L) = 1. \end{cases} \quad (13)$$

The country and the market take the institutional loan L as given. Differentiating U at the optimum with respect to L , and using the envelope theorem

$$dU = U_R dR + U_L dL.$$

Using the first equation in (13),

$$U_R = -\lambda(e)b, \quad U_L = \lambda(e)(R - R^*).$$

Hence,

$$dU = \lambda(e)(R - R^*)dL - \lambda(e)b dR. \quad (14)$$

Differentiating the equation of the lending rate (12),

$$0 = \lambda'(e)de \left(R - \frac{a - R^*L}{b} \right) + \lambda(e)dR - (1 - \lambda(e))\frac{R^*}{b}dL - (1 - \lambda(e))\frac{a - R^*L}{b^2}(dk - dL). \quad (15)$$

Using the first equation in (13), $dk = dR/f''$, and substituting in (14) the value of dR in (15), after some algebra, we find

$$dU = -(1 - \lambda(e))\frac{a - R^*L}{b}\frac{dR}{f''(k)} + \lambda'(e)b\frac{R - R^*}{1 - \lambda(e)}de, \quad (16)$$

and

$$(1 - \lambda(e))\frac{a - R^*L}{b} = R^* - \lambda(e)R. \quad (17)$$

Let us first consider the case where effort is exogenous and the probability of default, $1 - \lambda$ is fixed. The only endogenous variable is the level of investment (and therefore the amount of borrowing on the private market).

Exogenous effort

The welfare of the country is reduced to

$$U(b; R, L) = \lambda(a + f(L + b) - Rb - R^*L). \quad (18)$$

Using $\lambda'(e) = 0$ in (16) and (15), we find

$$\begin{cases} \left(\lambda b - (1 - \lambda) \frac{a - R^* L}{b} \frac{1}{f''} \right) dR = \lambda(R - R^*) dL, \\ dU = -(1 - \lambda(e)) \frac{a - R^* L}{b} \frac{dR}{f''(k)}. \end{cases} \quad (19)$$

In the first equation, the coefficients of dR and dL are both strictly positive. Hence, $dR/dL > 0$: an increase of the senior lending L , has a positive impact on the private rate R because of the higher risk of private loans. The higher private rate R induces a reduction of investment but that reduction of the demand for loans is only a second round effect and does not change the sign of the initial impact.

In the second equation, the coefficient of dR is positive. Therefore an increase of L , which generates an increase of R in the first equation, has a positive impact on the welfare of the borrowing country. Furthermore, assume that senior lending at the risk-free rate is at its maximum (to be risk-free), a/R^* . In the second equation of the system (19), the coefficient of dR is equal to zero: a small decrease of L has a first-order effect on the private rate R , as expected, but it has no first-order effect on the welfare of the country. We have the following result.

Proposition 2 (Seniority lending with endogenous investment and exogenous effort)

In the model of this section where investment by the borrowing country is endogenous and the resources for debt servicing are above a fixed level a , the level of senior lending L at the risk-free rate R^ can take any value in the interval $[0, a/R^*]$. The values of the rate R for other (junior) loans and of the welfare of the country are increasing functions of $L \in [0, a/R^*]$. The investment by the country is a decreasing function of L . At the maximum $L = a/R^*$, the variation of L has a zero first-order effect on the welfare of the country.*

We discuss this result after the analysis of the impact of L on effort.

Endogenous effort

We assume that the probability of default $1 - \lambda(e)$ is a decreasing function of effort. We consider only the case where $L = a/R^*$. Using $R^* = \lambda R$ in this case, the expression (16) becomes

$$dU = \lambda'(e)bRde. \quad (20)$$

At the maximum level of risk-free senior lending, the impact of L on welfare works through the impact on effort.

Differentiating the first-order condition for effort (13) and using the envelope theorem,

$$\lambda''(e)(f - Rb)de + \lambda'(e)\left((R - R^*)dL - b dR\right) = 0 \quad (21)$$

From the variation of the rate R in equation (15), we have

$$-b\lambda(e)dR = b\lambda'(e)Rde - (1 - \lambda(e))R^*dL. \quad (22)$$

Substituting in (21),

$$\lambda''(e)(f - Rb)de + \lambda'(e)\left((R - R^*)dL + b\frac{\lambda'(e)}{\lambda(e)}Rde - \left(\frac{1}{\lambda(e)} - 1\right)R^*dL\right) = 0$$

When $L = a/R^*$, $R^* = \lambda R$. Hence, in the previous equation $de = 0$. The variation of L has no first-order impact on effort and therefore no welfare impact.

Proposition 3 (Seniority lending and endogenous effort)

In the model of Proposition 2, when the effort that reduces the probability of default is endogenous, if the level of risk-free institutional lending is at the maximum a/R^ , a small reduction of the senior loan induces a smaller rate of the junior loans, and it has no impact on the level of effort and on the welfare of the borrowing country.*

The previous result relied on simple algebra but shows a local property near the maximum amount of senior lending at the risk-free rate. To complement this property, we analyze the impact of the senior loan over the entire range $[0, a/R^*]$ in a numerical example.

A numerical example

The functions that determine the level of resources and the probability of success are defined by

$$f(k) = Ak^\alpha, \quad \lambda(e) = 1 - \frac{\gamma}{1+e}$$

We take $R^* = 1$ and $a = 2$. We choose the value of γ such that if all the senior loan is issued against the safe resource, a/R^* , then the probability of default, $1 - \lambda$, is equal to 0.3. Note that

$$\frac{\gamma}{1+e} = 1 - \lambda. \quad (24)$$

Using (12), $R = R^*/\lambda$, and given R , the level of investment is equal to $k = \left(\frac{R}{\alpha A}\right)^{1/(1-\alpha)}$.

From the second equation in (13), we have

$$\frac{\gamma}{(1+e)^2}(a + f(k) - R(k - L) - R^*L) = 1.$$

By substitution and using (24),

$$e = (1 - \lambda)(a + f(k) - R(k - L) - R^*L) - 1, \quad \text{and} \quad \gamma = (1 + e)(1 - \lambda).$$

For each value of $L < a/R^*$, the equilibrium values of the loan rate, R , the investment by the country, k , and its effort e are determined by the equations (12) and (13). The welfare level, U , is then obtained from (18).

The values of R , e and U are represented in Figure 1. One verifies that these three values are increasing when the level of senior lending at the risk-free rate increases up to its maximum value that is equal to 2 in this example.

Discussion

One should emphasize the opposite impacts on the private lending rate and the probability of default. If the institution increases its lending with a seniority clause, the private lending rate increases because private loans cannot recover the safe part of the resources of the country and are more risky. That effect is the same as in the previous

Figure 1: Utility, interest rate and effort as a function of the senior lending

The maximum for the senior loan at the risk-free rate is 2. Welfare is scaled up to a constant to fit in the figure. The rate includes the repayment of the principal.

case with exogenous default. However, when the country can reduce the probability of default through effort, the higher private rate induces the country to make more effort and the welfare of the country is higher.

Official programs with new senior loans have obviously a negative impact on the price of *old* private loans that are junior and they increase the rate of these loans. But the previous proposition shows that when the official loan increases the rate on *new* private loans, that is not necessarily ground for criticism nor indication of a higher probability of default of the country. To the contrary, the result shows that the structure between senior and junior loans may induce a lower probability of default.

The result of Proposition 2 could be extended to senior loans that take some risk but a formal proof should be the subject of another paper. This result supports the concept of “Blue Bonds” that have been proposed (Delpla and Weizsäcker, 2011). The main idea is that countries could borrow up to a ceiling (of, say, 60% of GDP), by issuing Blue Bonds⁹, that would have seniority. Loans in excess of the ceiling would be obtained in the private markets and have junior status with a higher rate.

3 Multiple equilibrium interest rates

Can a country (*e.g.*, Italy, Ireland) face a high cost of debt because it has a higher probability of default, and can that higher probability be induced by a self-fulfilling high rate, *ex ante*?

In a standard debt contract, as in the model of the previous section, there may be multiple values of the loan rate that generates an *ex ante* return equal to the risk-free rate. When the contract is made between two parties, they can coordinate on the lowest rate, which is the most efficient in the class of debt contracts. In international sovereign lending with multiple lenders and contracts staggered over time, the coordination to establish the most efficient contract may be more difficult to achieve. We discuss this issue with a simple model.

The model

Consider first a two-period setting where the country borrows a quantity normalized to 1 in the first period. The loan is paid back in the second period in which there are three possible states of nature $j \in \{0, 1, 2\}$, with probability π_j that is common knowledge. In state j , the country’s income that can be used for debt payment is y_j , and $y_0 < y_1 < y_2$.

We assume that the lending contract has a fixed rate and that the country defaults

⁹ We ignore here the critical issue of the support by a central bank. A comprehensive analysis of the concept of Blue Bonds is beyond the present paper.

when its income is less than the loan rate. The bankruptcy causes a loss of resources that can be used for debt payment by a fraction κ . If the country defaults in state j , creditors receive $(1 - \kappa)y_j$. It is equivalent, under some specification, to assume that the country defaults when the cost of default is smaller than some penalty¹⁰. Without loss of generality, the country does not default in the highest state. Let R_j , $j \in \{0, 1\}$, be the loan rate (including principal) if the country defaults in the state k with $k \leq j$. From the previous assumptions, we have

$$y_0 < R_0 < y_1 < R_1 < y_2. \quad (25)$$

By arbitrage, the expected return on a loan is equal to the risk-free rate R^* . The loan rates with default in state 0, R_0 , and default in states 0 and 1, R_1 , are therefore determined by

$$\begin{cases} R^* = (1 - \pi_0)R_0 + \pi_0(1 - \kappa)y_0, \\ R^* = (1 - \pi_0 - \pi_1)R_1 + \pi_0(1 - \kappa)y_0 + \pi_1(1 - \kappa)y_1. \end{cases} \quad (26)$$

Hence,

$$R_0 = \frac{R^* - \pi_0(1 - \kappa)y_0}{1 - \pi_0}, \quad R_1 = \frac{R^* - \pi_0(1 - \kappa)y_0 - \pi_1(1 - \kappa)y_1}{1 - \pi_0 - \pi_1}. \quad (27)$$

Using the left inequality in (25) and the expressions for R_0 and R_1 ,

$$\pi_0(1 - \kappa)y_0 + (1 - \pi_0)y_1 - \kappa\pi_1y_1 < R^* < \pi_0(1 - \kappa)y_0 + (1 - \pi_0)y_1. \quad (28)$$

The right-hand inequality (for the equilibrium with low interest rate) holds if the probability of a very bad state, π_0 , is sufficiently small. The left-hand inequality (for the high interest rate equilibrium) holds when both the probability of the middle state and the cost of default in that state are sufficiently high. The critical assumption in the inequality (28) is that in the middle state, there is a significant loss of the resources.

¹⁰ For example, if the default penalty is a fraction μ of income, the country defaults if $y < R/\mu$.

Proposition 4

Consider a country that borrows one unit to be paid in the next period according to the model of this section with three states for the level of income in the next period. There are two equilibria under inequality (28), when the loss of resources in the middle state is sufficiently high. In one equilibrium, there is default only in the state with the lowest income. In the other equilibrium, the loan rate is higher and the country does not default only in the state with the highest income.

Assume now that the country is initially in the bad equilibrium with rate R_1 and renegotiates a fraction m of its debt to be serviced at the rate $\tilde{R} = R_0$, that is the lower rate of the good equilibrium. The statutory debt service is now

$$B(m) = (1 - m)R_1 + mR_0.$$

Under the previous assumptions about the parameters of the model (for which there are two equilibria), we have

$$B(0) = R_1 > y_1, \quad B(1) = R_0 < y_1.$$

For some value $m^* \in (0, 1)$, if $m > m^*$, the country does not need to default in state 1 on the debt that earns a return R_1 . In other terms, the market value of that debt is greater than 1. The country could buy that debt back by issuing a new debt with a return R_0 .

Proposition 5

Assuming two equilibria in the model of Proposition 4, there is a value m^ , $0 < m^* < 1$, such that if the country refinances a fraction of its private debt $m \geq m^*$ at the low rate R_0 , then it is able to refinance the whole debt at the rate R_0 and switch to the good equilibrium at the rate R_0 .*

The previous analysis shows that if the country is in a bad equilibrium with a high rate, a switch to the good equilibrium requires substantial refinancing. Here the situation

of a country that faces a large number of creditors is different from that of a private firm that faces one or two large creditors. There is a coordination issue between the country and its lenders.

One could argue that the coordination could be solved by sequential negotiations between the country and its creditors, of finite number N , along the lines of the argument of Gale (1995). Begin with $N = 2$. If the country and the first creditor come to an agreement at the lower equilibrium rate R_0 , then in the next step, the remaining creditor and the country establish a loan contract at the same rate R_0 . In the first step, the country and the first creditor know the outcome of that sub-game in the second step. That knowledge sustains the equilibrium rate R_0 in the first step¹¹. The argument can be generalized to any number N .

There are a number of difficulties with that argument. First, the argument assumes cost-free negotiations but negotiations take time and are costly. In the 18th century, the English government faced that issue. Expectations had shifted toward a low rate and the old loans, at the high rate, were traded significantly above par. Although these loans were redeemable and the government could legally repurchase them by issuing new bonds at a lower rate, that operation was costly. This setting of a “lowering the interest rate” of the public debt generates a coordination game with multiple equilibria (success and failure of refinancing) that was analyzed in Chamley (2011). The English government failed in its first attempt in 1737 because it did not provide sufficient incentive for bond holders to coordinate on the refinancing equilibrium. The government succeed in 1749 when it raised the premium for subscribing to bonds with a lower coupon. The high cost of issuing debt and refinancing may be not smaller today, as shown for private issues by Chen and Ritter (2000) and Abrahamson *et al.* (2011).

¹¹ In the model of Gale, the situation is slightly different: with $N = 2$, there are two agents, each with an option to make a unit investment, at some cost. The dividend is a flow that begins in the period of the investment with a present value greater than the cost only if both agents have invested. A vanishingly short period solves the coordination problem and the result can be extended by induction to any N finite. However, the analysis shows that issues of timing are critical for the properties of the model.

Second, the argument about sequential negotiations assumes common knowledge on the structure of the game. If at some stage, there is uncertainty on the next stage of negotiation, one can conjecture that the uncertainty operates as the cost of negotiation in the previous paragraph. A formal analysis is beyond the scope of the present paper.

Proposition 5 provides a *caveat* on the benefits of the long debt maturity that were presented by Giavazzi and Pagano (1989). In their argument, a long maturity enables the country to reduce the amount that is due in each period, thereby lowering the risks associated with a temporary crisis of liquidity or confidence. Consider a country that faces a high rate on its debt because it has gone through difficult times in the past and suppose that current conditions are much improved. From the previous result, if it refinances its debt gradually as the old bonds come to maturity, the high rate may be the only equilibrium rate. The switch to the better equilibrium requires a quantum refinancing of the debt at a lower rate. Of course, an institution could provide a bridge loan to facilitate that transition.

The impact of senior institutional lending

Suppose all agents coordinate on the equilibrium with higher interest rate and the country defaults in states 0 and 1, in the second period. Assume that an institution such as the IMF lends m at the risk-free rate R^* , with senior claim on repayment, and that in the lowest state, with income y_0 , the country has sufficient resources to pay off the IMF loan (which can therefore be granted at the risk-free rate): $y_0 > mR^*$.

Assume that the country defaults in states 0 and 1. The loan rate, R_1 , is such that

$$R^*(1 - m) = (1 - \pi_0 - \pi_1)R_1(1 - m) + \pi_0((1 - \kappa)y_0 - mR^*) + \pi_1((1 - \kappa)y_1 - mR^*).$$

Note that on the right-hand side, the payment to the IMF has seniority and is deducted from the available resources y_i , ($i = 0, 1$) before any resource is transferred to private creditors. The cost of default applies to this transfer but not to the transfer to the IMF.

Straightforward algebra shows that the rate R_1 on the private loans that is required

for an *ex ante* return R^* is given by.

$$R_1 = \frac{R^*}{1 - \pi_0 - \pi_1} - \frac{1}{1 - \pi_0 - \pi_1} \left(\pi_0 \frac{(1 - \kappa)y_0 - mR^*}{1 - m} + \pi_1 \frac{(1 - \kappa)y_1 - mR^*}{1 - m} \right).$$

As in the previous section, that rate increases with the amount of the IMF loan because loans by the private sector are junior to the IMF loan and are more risky. The statutory debt service, b , is equal to

$$b = R_1(1 - m) + R^*m = \frac{R^*}{1 - \pi_0 - \pi_1} + \frac{-1}{1 - \pi_0 - \pi_1} \left(\pi_0((1 - \kappa)y_0 + \pi_1(1 - \kappa)y_1) \right).$$

The value is independent of the IMF loan. The property is not surprising and is just an application of Proposition 1. The IMF loan at the risk-free rate is neutral, *per se*. The intervention of the IMF may trigger a switch to the equilibrium with a lower rate (and default in state 0 only), but this effect is independent of the amount of the loan. Note that if the IMF would make a loan at a rate lower than the risk-free rate, it would lower the statutory payment and that subsidy, if sufficiently large, may trigger the switch to the better equilibrium.

Proposition 6

If there are two equilibria for new loans in the model of Proposition 4, an institutional loan with seniority at the risk-free rate does not alter the existence of these two equilibria.

Throughout this section, we have assumed that the probability distribution of resources of the borrowing country is exogenous. The case of endogenous effort is beyond the scope of this paper but, given the results of the previous section, we can suggest the following remark. Seniority of official lending increases the rate of other loans, has a positive impact on the effort of the borrowing country and increases its welfare (Proposition 3). Suppose that in the framework of the present section, the country can reduce through effort the probability of the middle state, with resources y_1 , that the inequalities (28) hold for some levels of effort, and that the country is in the bad equilibrium with default in the middle state. A restructuring of the debt with a senior official loan and a higher rate on private loans induces more effort. The

effect may reduce the probability of the middle state to invalidate the left-hand side in the inequality (28) and cancel the bad equilibrium. Under these conditions, official lending with seniority could generate a shift from a bad to a good equilibrium with a lower debt service burden. Because the jump would not be on the margin, the welfare improvement would be of an order of magnitude higher than in Proposition 3.

4 Seniority and old loans

In the previous sections, we focused on the interaction between senior institutional loans and new private loans. However, all institutional interventions take place only after a shock puts the economy in a state that was different from the previous expectations of private creditors. As mentioned in Section 2 and as shown by recent events in Europe, new loans with seniority may have a negative impact on the value of old loans. Should old loans absorb some of the negative shocks through some debt forgiveness, or should new loans be used to service old loans, in part or in full? We analyze this issue here in a simple model. For simplicity, we ignore the impact of a contingent debt forgiveness in period 1 on the expectations for the rate of the loans written before that period.

Assume first that because of a shock, the country needs to raise the amount z in period 1. The country has a random amount of resources y in period 2, net of the payments for previous debt commitments, that can be used to service the new loans. The financing of z is done through the sum of a reduction in the payment on old loans (debt forgiveness), by the amount x , and new loans equal to $z - x$. Since we focus on the sharing of the shock between old and new loans, we ignore the issue of seniority for new loans, and we assume that the distribution of y is exogenous. Following Proposition 1, we can assume that all new loans have equal seniority and have the same rate of return R .

If a default takes place, there are in general two costs, which are borne by the country and the lenders, respectively. We ignore the cost borne by the country and assume that the lenders can claim what is available in the country minus a cost that increases with

the level of the default, that is the due payment minus the available resources. That increasing property is plausible. The cost of debt reduction is therefore a particular case of the cost function τ in Section 2: it is an increasing function $\kappa(\cdot)$ of the amount of the debt reduction with

$$\kappa(0) = 0, \quad \kappa'(x) > 0, \quad \kappa''(x) \geq 0. \quad (29)$$

The function κ does not need to be strictly convex.

In period 2, the country defaults if the income y is less than the payment due $R(z-x)$. Note that if there is no new loan (and all the shock is absorbed by the old loans in period 1), there is no default in period 2. The value of R is determined by arbitrage with risk-free investment:

$$\begin{cases} R^*(z-x) = (z-x)R \int_b^1 f(y)dy + \int_b^1 (y - \kappa((z-x)R - y))f(y)dy, \\ \text{with } b = (z-x)R. \end{cases} \quad (30)$$

On the right-hand side of the previous equation, the first term is the payment in period 2 with no default. The second term is the payment under default. The solution in R is a function of the debt reduction x , in period 1.

We consider the total cost of all debt reductions in periods 1 and 2. These costs may be borne by different agents but from a social point of view, we just add them up. (We assume no interaction between the costs in periods 1 and 2.) The total cost of debt reduction is therefore a function of the debt reduction in period 1.

$$C(x) = \kappa(x) + \frac{1}{R^*} \int_b^1 \kappa(b-y)f(y)dy, \quad (31)$$

where $b = (z-x)R$, and the rate R is a function of x in equation (30). By assumption, the cost of default in period 2 is discounted to period 1 at the risk-free rate.

In order to evaluate the impact of the debt reduction x on the cost C , differentiate equation (30) to determine the variation of the rate R :

$$\begin{aligned} R^*dx &= db \left(\int_b^1 \kappa'(b-y)f(y)dy - \int_b^1 f(y)dy \right) \\ &\quad + Rdx \left(bf(b) - (b - \kappa(0))f(b) \right), \quad \text{with } db = (z-x)dR - Rdx. \end{aligned}$$

Using $\kappa(0) = 0$,

$$R^* dx = \left(\int^b \kappa'(b-y)f(y)dy - \int_b f(y)dy \right) db. \quad (32)$$

Differentiating (31), and using $\kappa(0) = 0$,

$$dC = \kappa'(x)dx + \frac{db}{R^*} \int^b \kappa'(b-y)f(y)dy.$$

Substituting (32) for the substitution of db ,

$$C'(x) = \kappa'(x) - \frac{\int^b \kappa'(b-y)f(y)dy}{\int_b f(y)dy - \int^b \kappa'(b-y)f(y)dy}. \quad (33)$$

We consider two cases for the cost function $\kappa(\cdot)$. First, assume that the cost of default κ is an increasing and convex function such that $\kappa'(0) = \kappa(0) = 0$. In this case, $C'(0) < 0$ in (33). It is then obvious that some debt reduction in period 1 is efficient. The property is equivalent to the “smoothing” of the cost of default between periods (like the tax smoothing property in efficient inter-temporal taxation).

Second, assume that the marginal cost is constant and equal to α . The previous expression becomes

$$C'(x) = \alpha \left(1 - \frac{\pi}{1 - \pi - \alpha\pi} \right),$$

where π is the probability of default in period 2, $\pi = \int^b f(y)dy$.

An increase of the debt forgiveness, x , in period 1 generates a smaller total cost of debt reduction if and only if

$$\pi > \frac{1}{2 + \alpha}.$$

Note that the default takes place for $y < (z - x)R$, with R determined in (30). Therefore, the probability of default π is a decreasing function of the debt reduction, $\pi(x)$. No debt reduction in period 1 is efficient if $\pi(0) < 1/(2 + \alpha)$. The previous discussion is summarized by the following proposition.

Proposition 7

Consider a country that has a debt payment due in period 1 when a shock requires new financing.

(i) If the cost $\kappa(\zeta)$ of a debt reduction ζ is such that $\kappa(0) = \kappa'(0) = 0, \kappa''(\zeta) \geq 0$, some debt forgiveness is efficient in period 1.

(ii) If the marginal cost of debt reduction is constant and equal to α , some debt reduction in period 1 is efficient if and only if when no reduction takes place, the probability of default in period 2 is greater than $1/(2 + \alpha)$.

This proposition is related to the results of Conesa and Kehoe (2011). When the probability of default in period 2 is not too high, it pays to “gamble for redemption”. Rather than paying for the default in period 1, the country should take the gamble to wait until period 2 for a favorable outcome in the level of resources, y .

5 Conclusion

According to the official policy of the IMF, “the IMF, when it lends, provides only a small portion of a country’s external financing requirements,” and “IMF financing can act as an important lever, or catalyst, for attracting other funds” (IMF web site). That effect operates through two channels. In the first, the IMF lending induces the country to undertake reforms that increase future growth and the profitability of investment. In the second, it has an impact on the credit market. The analysis in this paper suggests that the seniority of the IMF lending with respect to new loans, although it increases the interest rate that the country faces on the margin in the financial markets, may enhance the incentive for efficient investment and reform in the borrowing country. This positive impact is taken into account by the financial market. Although the rate in that market rises, the overall effect on the welfare of the country is positive.

We bring some additional elements for the analysis of the seniority of new loans with

respect to old loans. Whether a haircut on old loans is efficient depends on the relative costs of default in the present and in the future and the probability of default in the future. If the cost of default is small and the probability of default in the future is small, then our analysis supports the argument of Conesa and Kehoe (2011) in which an immediate haircut is avoided through gambling for a favorable outcome in the future. However, as mentioned at the beginning of this paper, recent events suggest that these conditions are not met, in which case the argument of Chamley and Pinto (2011) applies: immediate debt reduction reduces the overall cost of bailouts.

In further work, we hope to extend the analysis by including multiple periods. As we have suggested in Section 3, that context may be particularly important for the role of seniority in the catalytic effect during liquidity crises.

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