Reinsurance as Capital Optimization Tool under Solvency II

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Abstract

This paper compares solvency capital requirements under Solvency I and Solvency II for a sample mid-size insurance portfolio. According to the results of a study, changing the solvency capital regime from Solvency I to Solvency II will lead to a substantial additional solvency capital requirement that might represent a heavy burden for the company's shareholders.

One way to reduce the capital requirement under Solvency II is to increase reinsurance protection, which will reduce the net retained risk exposure and hence also the solvency capital requirement. Therefore, this paper proposes an extended reinsurance structure that, under Solvency II, brings the capital requirement back to the level of that required under Solvency I. In a step-by-step approach, the paper demonstrates the extent of solvency

relief attained by the insurer by applying different possible adjustments in the reinsurance structure.

To evaluate the efficiency of reinsurance as the solvency capital relief instrument, the authors introduce a cost-of-capital based approach, which puts the achieved capital relief in relation to the costs of extending the reinsurance protection. This approach allows a direct comparison of reinsurance as a capital relief instrument with debt instruments available in the capital market. With the help of the introduced approach, the authors show that the best capital relief efficiency under all examined reinsurance alternatives is achieved when a financial quota share contract is chosen for proportional reinsurance.

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1. Introduction

For a sample insurance portfolio, this paper compares the solvency capital requirements under Solvency I and Solvency II (QIS5 Standard Formula), with the latter taking into account only two risk categories: non-life underwriting risk and counterparty default risk due to reinsurance³. The results of this comparison are presented in Section 2.

In Section 3, the paper shows that under Solvency II substantially more extensive reinsurance protection is required to achieve the same minimum level of solvency capital requirements as under Solvency I which results in considerably higher costs of reinsurance. We show that these costs can be reduced only partially by changing the structure of reinsurance coverage. In Section 3 we also introduce the concept of financial quota share reinsurance as a powerful tool for reducing reinsurance costs when reinsurance is used as a solvency relief instrument. Using the capital cost of different instruments as a benchmark, we develop an approach for evaluating the efficiency of reinsurance as a substitute for solvency capital.

Section 4 presents the conclusions and summarizes the main results of the paper.

2. Solvency capital requirements under Solvency I and Solvency II for a sample insurance portfolio

2.1 Sample insurance portfolio and its reinsurance protection

We assume that the sample portfolio contains Motor Liability, Motor Hull, Marine, Fire and General Liability insurance business, cf. Table 2.1. Besides presenting gross premium per line of business, the table contains the estimates of the ultimate loss ratio⁴ and ultimate loss for last business year.

Table 2.1.1: Sample portfolio gross premium composition and ultimate losses⁵

Line of business	A. Gross Premium 2011	B. [%] Ultimate Loss Ratio 2011	C = A * B Ultimate gross Loss 2011
Motor vehicle liability insurance	20,000	70%	14,000
Other motor insurance	5,000	60%	3,000
Marine, aviation and transport insurance	5,000	80%	4,000
Fire and other damage to property insurance	15,000	70%	10,500
General liability insurance	5,000	60%	3,000
Total	50,000	69%	34,500

³ For the purposes of this comparison, we ignored such Solvency II risk categories as market risk, operational risk and default risk due to counterparties other than reinsurance as these risk categories have no equivalents in the “old world” of Solvency I.

⁴ In this paper, the term “ultimate loss ratio” is used to describe the ratio between the ultimate underwriting loss under an annual contract period after its full development and the gross insurance premium written under the contract during the same annual term.

⁵ Throughout this paper, all figures presented in tables are denominated in thousands (€'000).

Further, we assume that the sample portfolio is protected by proportional reinsurance for such lines of business as Marine and Fire and by the non-proportional excess of loss reinsurance for all lines of business. The following table summarizes the parameters of proportional reinsurance for the sample portfolio.

Table 2.1.2: Proportional reinsurance for the sample portfolio

Line of business	D. Proportional RI cession	E = A * D Ceded Premium 2011 (prop. RI)	F = A - E Retained Premium 2011 (prop. RI)
Motor vehicle liability insurance	0%	0	20,000
Other motor insurance	0%	0	5,000
Marine, aviation and transport insurance	50%	2,500	2,500
Fire and other damage to property insurance	30%	4,500	10,500
General liability insurance	0%	0	5,000
Total	14%	7,000	43,000

The non-proportional XL (excess of loss) reinsurance is chosen in the way that the company's net retention due to proportional reinsurance is protected up to the level of retained Probable Maximum Loss (PML) for a selected return period with the self-retention of €500,000 for each line of business. Such self-retention represents a common retention choice for a portfolio of given size in emerging markets. The following table illustrates the approach and provides a summary of parameters for the company's XL reinsurance.

Table 2.1.3: Non-proportional reinsurance for the sample portfolio

Line of business	G. PML gross	H = G * (1 - A) PML net retained after prop. RI	I. Non-prop XL RI 2011
Motor vehicle liability insurance	10,000	10,000	9,500 xs 500
Other motor insurance	10,000	10,000	9,500 xs 500
Marine, aviation and transport insurance	50,000	25,000	24,500 xs 500
Fire and other damage to property insurance			
Fire non natural catastrophe per risk	30,000	21,000	20,500 xs 500
Fire natural catastrophe per event (EQ)	100,000	70,000	69,500 xs 500
General liability insurance	5,000	5,000	4,500 xs 500

Please note that under the line of business Fire along with the “pure” fire exposure there is also the natural catastrophe exposure due to earthquake that requires reinsurance cover. This necessitates two XL reinsurance contracts for this line of business – one for the non-natural-catastrophe per risk exposure and one for the earthquake per event exposure.

According to Solvency I, cf. Section 2.2 below, for calculating solvency capital requirements the total gross and net retained losses for the last three years are required. The following table calculates these parameters assuming (a) that the loss burdens per line of business for the years 2009 and 2010 were equal to the ones of the year 2011 and (b) that within last three years the non-proportional reinsurance contracts paid out €500,000 each.

Table 2.1.4: Total gross and net underwriting loss for the last three business years

Line of business	K = 3 * C Ultimate gross loss 2009- 2011	L = K * (1 - D) Ultimate net loss retained under prop. RI 2009-2011	M Ultimate loss covered by non-prop. RI 2009-2011	N = L - M Ultimate net loss retained under prop. & non- prop RI 2009-2011
Motor vehicle liability insurance	42,000	42,000	500	41,500
Other motor insurance	9,000	9,000	500	8,500
Marine, aviation and transport insurance	12,000	6,000	500	5,500
Fire and other damage to property insurance	31,500	22,050	1,000	21,050
General liability insurance	9,000	9,000	500	8,500
Total	103,500	88,050	3,000	85,050

2.2 Solvency capital requirements for the sample insurance portfolio according to Solvency I⁶

First, the Retention Ratio needs to be calculated. The following table provides the result for the sample portfolio whereas the figures in columns A and B were taken from the Table 2.1.4, columns K and N.

Table 2.2.1: Solvency I, Retention Ratio

A. Total gross loss of last three	B. Total net loss of last three years	C = B / A Retention ratio
103,500	85,050	82%

Then we can calculate the Premium Index (Table 2.2.2) and the Claims Index (Table 2.2.3)

Table 2.2.2: Solvency I, Premium Index (EUR 000)

D. Adjusted gross premium 2011	E. Adjusted gross premium under € 57.5m	F. Adjusted gross premium above € 57.5m	G = C * (0.18 * E + 0.16 * F) Premium Index
52,500	52,500	0	7,765

Adjusted gross premium in column D of Table 2.2.2 is computed as the total gross premium plus 50% of the premium for general liability, cf. Table 2.1.1 and Annex A1.

Table 2.2.3: Solvency I, Claims Index (EUR 000)

H. Adjusted gross loss 2009-2011	I = H / 3 Mean adjusted gross loss of 2009-2011	J. Mean adjusted gross loss under € 40.3m	K. Mean adjusted gross loss above € 40.3m	L = C * (0.26 * J + 0.23 * K) Claims Index
108,000	36,000	36,000	0	7,691

The adjusted gross loss in column H of Table 2.2.3 is the product of the total gross loss plus 50% of the loss for general liability, cf. Table 2.1.4 and Annex A1.

⁶ For more details on calculating solvency capital requirements under Solvency I, see Annex A1.

The solvency capital requirement according to Solvency I is calculated as the maximum between the Premium Index and Claims Index. For the sample portfolio, we obtain

$$\begin{aligned}\text{Solvency Capital Requirement} &= \max (\text{Premium Index; Claims Index}) \\ &= \max (7,765; 7,691) = \text{€}7,765\end{aligned}$$

2.3 Solvency capital requirements for the sample insurance portfolio under Solvency II

This section provides the results of capital requirement calculation under Solvency II for such risk categories as non-life underwriting risk and counterparty default risk due to reinsurance. More details of the capital requirement calculation are provided in Annexes A2 and A3.

In our calculation, for simplification purposes we ignored such Solvency II risk categories as market risk, operational risk and default risk of counterparties, other than that of reinsurers, as these Solvency II risk categories have no equivalent under the “old world” of Solvency I.

The capital requirement for non-life underwriting risk is the total of such sub-modules as non-life premium and reserve risk, lapse risk sub-module and catastrophe risk. For the purposes of our calculation, we assumed that the lapse risk can be neglected. Furthermore, we assumed that the sub-module man-made catastrophe risk can be neglected as well. Therefore, under the risk category for non-life underwriting risk, only the sub-modules for premium and reserve risk and natural catastrophe risk need to be considered.

2.3.1 Capital requirement for premium and reserve risk

For calculating the capital requirement for the sub-module premium and reserve risks, we first calculate the net retained premium and reserves. The net retained premium is calculated as the gross premium income net of reinsurance premium. The gross premium and the premium retained after proportional reinsurance for the previous year (2011) were already provided in Section 2.1, Table 2.1.2. We assume that for the next year (2012) these premiums are equal to the ones of 2011. Table 2.3.1 provides the 2012 reinsurance premiums for non-proportional reinsurance and finally the net premium retained under both proportional and non-proportional reinsurance. For non-proportional reinsurance, we assumed that in 2012 the sample portfolio is protected by the same XL contracts as in 2011.

Table 2.3.1: XL reinsurance premium and net retained premium

Line of business	A. Retained Premium (prop RI) 2012	B. Non-prop XL RI 2012	C. Non-prop XL RI Rate on GNPI	D = A * C XL RI premium 2012	E = A - D Retained Premium (prop & XL) 2012
Motor vehicle liability insurance	20,000	9,500 xs 500	2.3%	460	19,540
Other motor insurance	5,000	9,500 xs 500	4.9%	247	4,753
Marine, aviation and transport insurance	2,500	24,500 xs 500	9.6%	240	2,260
Fire and other damage to property insurance	10,500				8,248
Fire non natural catastrophe per risk		20,500 xs 500	14.6%	1,530	
Fire natural catastrophe per event (EQ)		69,500 xs 500	6.9%	722	
General liability insurance	5,000	4,500 xs 500	4.4%	222	4,778
Total	43,000			3,421	39,579

Reinsurance rates on premium in Column C of Table 2.3.1 were calculated based on market benchmarks for corresponding loss frequencies and loss severities (see Annex A4 for more

details). These are gross rates, i.e. they cover both the expected layer loss as well as the reinsurer's costs of capital and administrative expenses allocated to the XL layer.

Along with the net retained premium, one also needs to estimate the provisions for outstanding claims for calculating capital requirements in the premium and reserve risk sub-module (see Annex A2). Table 2.3.2 summarizes our assumptions regarding the provisions for outstanding claims.

Table 2.3.2: Provisions for outstanding claims

Line of business	F. Gross provisions for outstanding claims	G. Net retained provisions for outstanding claims
Motor vehicle liability insurance	15,000	13,000
Other motor insurance	2,000	1,500
Marine, aviation and transport insurance	1,000	400
Fire and other damage to property insurance	1,000	500
General liability insurance	3,000	2,700
Total	22,000	18,100

Assumptions for gross provisions in Column F were made based on the gross premium (Table 2.1.1, Column A) and the characteristic payout patterns for each line of business. Net retained provisions in Column G were assumed based on the gross provisions in column F and the risk mitigating effect of both proportional and non-proportional reinsurance.

The first step in calculating the capital requirement for premium and reserve risks is to calculate the volume measures for premiums and reserves, cf. Table 2.3.3.

Table 2.3.3: Solvency II, premium and reserve risk, volume measures

	H = A Volume measure premium	I = G Volume measure reserves	J = H + I Volume measure V _{lob}
Motor vehicle liability insurance	19,540	13,000	32,540
Other motor insurance	4,753	1,500	6,253
Marine, aviation and transport insurance	2,260	400	2,660
Fire and other damage to property insurance	8,248	500	8,748
General liability insurance	4,778	2,700	7,478
Total	39,579	18,100	57,679

The next step is to calculate standard deviations per line of business, cf. Table 2.3.4. This calculation takes into account the NP factors (column L) which represent the volatility reduction effect of non-proportional reinsurance.

Table 2.3.4: Solvency II, premium and reserve risk, standard deviations

	K. StdDev _{prem} without NP Factors	L. NP Factors	M = K * L StdDev with NP Factors	N. StdDev _{res}	P. StdDev _{lob}
Motor vehicle liability insurance	10.0%	74.7%	7.5%	9.5%	7.2%
Other motor insurance	7.0%	72.1%	5.0%	10.0%	5.4%
Marine, aviation and transport insurance	17.0%	60.3%	10.3%	14.0%	9.9%
Fire and other damage to property insurance	10.0%	60.6%	6.1%	11.0%	6.1%
General liability insurance	15.0%	77.4%	11.6%	11.0%	10.0%

Table 2.3.5 completes the calculation. It provides the results for the total volume measure (column Q), total standard deviation (column R), the function ρ of the total standard deviation, which approximates the 99.5% VaR (column S), and finally the capital requirement for non-life premium and the reserve risk (column T).

Table 2.3.5: Solvency II, capital requirement for premium and reserve risk

	Q = total of J Total volume measure V	R. Total StdDev	S. rho (std_dev)	T = Q * S Capital requirement prem & reserve risk NL_pr
Total	57,679	5.7%	15.7%	9,044

For a detailed explanation of the calculation steps presented in Tables 2.3.3, 2.3.4 and 2.3.5, please refer to Annex A2, section A2.2.

2.3.2 Capital requirement for natural catastrophe risk

The only natural catastrophe risk covered by the sample portfolio is the earthquake risk. According to Table 2.1.3, the PML for the earthquake risk is € 100m. We assume that this PML is the 200 years loss. The capital requirement for earthquake risk according to Solvency II is equal to the 200 years loss net of the risk mitigation due to reinsurance. The following Table 2.3.6 summarizes the calculation of risk mitigation and provides the resulting capital requirement.

Table 2.3.6 Capital requirement for natural catastrophe risk

Peril	A. Scenario gross	B. Risk Mitigation prop RI	C. Risk Mitigation XL RI	D. Reinstatement premium XL RI	E = B + C - D Overall Risk Mitigation	F = A - E SCR / peril	G = total F (diversified) SCR
Windstorm	0				0	0	
Earthquake	100,000	30,000	69,500	722	98,778	1,222	
Flood	0				0	0	
Hail	0				0	0	
Subsidence	0				0	0	
Total							1,222

For more details on the calculation of capital requirement for natural catastrophe risk, please refer to Annex A2, section A2.4.

2.3.3 Total capital requirement for premium and reserve risk and natural catastrophe risk

In this section, we calculate the total capital requirement for premium and reserve risks and the natural catastrophe risk. Capital requirements for these sub-modules taken individually were calculated in sections 2.3.1 and 2.3.2. The total capital requirement results from both individual capital requirements and some diversification effects, cf. Annex A2, section A2.1.

Table 2.3.7 provides the resulting capital requirement.

Table 2.3.7 Total capital requirement for premium, reserve risk and natcat risk

	NL SCR
A. NL premium & reserve	9,044
B. NL Natural catastrophe	1,222
C = A + B: Simple total	10,266
D. Diversification effect	842
E = C - D: Total SCR	9,424

2.3.4 Capital requirement for counterparty credit default risk from reinsurance

According to Annex A3, *LGD* (loss given default) for each reinsurance contract is calculated as

$$LGD_i = \max((1 - RR) \cdot (Recoverables_i + RM_i); 0)$$

where

RR = Recovery rate for reinsurance arrangements. According to SCR6.52, for reinsurance arrangements, $RR = 50\%$ can be assumed.

$Recoverables_i$ = Best estimate recoverables from the reinsurance contract i . We assume that $Recoverables_i$ = expected loss ceded to the reinsurance contract i plus reinsurance share on provisions for outstanding claims.

According to SCR6.17 and SCR6.32, *Risk Mitigation* RM_i can be calculated as

$$RM_i = (SCR_{gross} - SCR_{net}) \cdot \frac{Recoverables_i}{Recoverables_{total}}$$

where

SCR_{gross} = solvency capital requirements for underwriting risk calculated without reinsurance

SCR_{net} = solvency capital requirements for underwriting risk taking reinsurance into account

Therefore, for calculating *LGD* for each counterparty and then the capital requirement for counterparty credit default risk, we first need to calculate the recoverables and risk mitigation.

The following Table 2.3.8 provides the results for the expected loss ceded to proportional reinsurance.

Table 2.3.8: Expected loss ceded to proportional reinsurance

Line of business	A. Expected Gross Premium 2012	B. [%] Expected Ultimate Loss Ratio 2012	C. Proportional RI cession	D = A * B * C Expected loss 2012 ceded to prop RI
Motor vehicle liability insurance	20,000	70%	0%	0
Other motor insurance	5,000	60%	0%	0
Marine, aviation and transport insurance	5,000	80%	50%	2,000
Fire and other damage to property insurance	15,000	70%	30%	3,150
General liability insurance	5,000	60%	0%	0
Total	50,000			5,150

The next step is to calculate the expected loss ceded to non-proportional reinsurance. This calculation is carried out in Table 2.3.9. Further, this table (column I) provides the result for the total expected loss ceded to both proportional and non-proportional reinsurance.

Table 2.3.9: Expected loss ceded to XL reinsurance and to both proportional and XL reinsurance

Line of business	E. Retained Premium (prop RI) 2012	F. Non-prop XL RI 2012	G. Non-prop XL RI Net rate on GNPI	H = E * G Expected loss 2012 ceded to XL RI	I = D + H Expected loss 2012 ceded to prop & XL RI
Motor vehicle liability insurance	20,000	9,500 xs 500	1.5%	306	306
Other motor insurance	5,000	9,500 xs 500	3.3%	165	165
Marine, aviation and transport insurance	2,500	24,500 xs 500	6.4%	160	2,160
Fire and other damage to property insurance	10,500				4,531
Fire non natural catastrophe per risk		20,500 xs 500	9.7%	1,020	
Fire natural catastrophe per event (EQ)		69,500 xs 500	3.4%	361	
General liability insurance	5,000	4,500 xs 500	3.0%	148	148
Total	43,000			2,160	7,310

As explained above, the reinsurance recoverable consists of the expected ceded loss and reinsurance share on provisions for outstanding claims. The expected ceded loss has been calculated in the Tables 2.3.8 and 2.3.9. Table 2.3.10 provides the results for claims provisions and the results for reinsurance recoverables (column M).

Table 2.3.10: Reinsurance share on provisions for outst. claims and reinsurance recoverables

Line of business	J. Gross provisions for outstanding claims	K. Net retained provisions for outstanding claims	L = J - K Reinsurance share on provisions for outst. claims	M = I + L Reinsurance recoverables
Motor vehicle liability insurance	15,000	13,000	2,000	2,306
Other motor insurance	2,000	1,500	500	665
Marine, aviation and transport insurance	1,000	400	600	2,760
Fire and other damage to property insurance	1,000	500	500	5,031
General liability insurance	3,000	2,700	300	448
Total	22,000	18,100	3,900	11,210

Apart from reinsurance recoverables, Loss Given Default also includes a Risk Mitigation component, which is calculated in Table 2.3.11.

Table 2.3.11: Risk Mitigation

N. NL capital requirement with reinsurance	O. NL capital requirement without reinsurance	P = O - N Risk mitigation
9,424	104,179	94,755

We further assume that the reinsurance panel is made up of four reinsurance companies, each taking one fourth of the total reinsurance program and having an A rating. Table 2.3.12 provides the calculation results for the Loss Given Default for each of four reinsurance companies, please refer to Annex A3 for more details.

Table 2.3.12: Reinsurance recoverables, Risk Mitigation and Loss Given Default per company

Counterparty	Q = M / 4 Reinsurance recoverables	R = P * Q / total Q Risk mitigation	S. Recovery rate	T = (1 - S) * (Q + R) Loss Given Default
Reinsurance company 1	2,803	23,689	50%	13,246
Reinsurance company 2	2,803	23,689	50%	13,246
Reinsurance company 3	2,803	23,689	50%	13,246
Reinsurance company 4	2,803	23,689	50%	13,246
Total	11,210	94,755		52,983

Applying the capital requirement calculation methodology explained in Annex A3 to the results for Loss Given Default provided in Table 2.3.12 results in the capital requirement provided in Table 2.3.13.

Table 2.3.13: Capital requirement for counterparty credit default risk from reinsurance

	SCR
Default	2,635

2.3.5 Total capital requirement for non-life underwriting risk and credit default risk

In this section, we calculate the total capital requirement for non-life underwriting risk and counterparty credit default risk from reinsurance. Taken individually, capital requirements for these sub-modules were calculated in sections 2.3.3 and 2.3.4. The total capital requirement takes into account both individual capital requirements and partial diversification effects (see Annex A2, section A2.5). Table 2.3.13 provides the resulting capital requirement.

Table 2.3.13: Total capital requirement for non-life underwriting risk and credit default risk

	SCR
A. NL underwriting risk	9,424
B. Deafult	2,635
C = A + B: Simple total	12,059
D. Diversification effect	1,078
E = C - D: Total SCR	10,981

2.3.6 Comparison of capital requirements according to Solvency I and Solvency II

In sections 2.2 and 2.3, capital requirements for the chosen sample portfolio under Solvency I (€ 7.8m) and Solvency II (€ 11.0m) were calculated. As can be seen, the Solvency II capital

requirement is more than 40% higher than that of Solvency I which might represent a heavy burden for the company's shareholders. One of the ways to deal with the increased capital requirement is to increase the cession to reinsurance which will reduce the net retained risk exposure and hence the capital requirements. However, this will also increase the costs of reinsurance if no restructuring of reinsurance portfolio is undertaken. In the next section 3, we derive the reinsurance structure which will bring the Solvency II capital requirement to the level of that for Solvency I, discuss different alternatives for the reinsurance structure and compare the resulting costs of reinsurance.

3. Reinsurance portfolio for reducing Solvency II capital requirement to Solvency I level

As demonstrated in Section 2, the capital requirement for premium and reserve risk accounts for the largest "portion" of the overall capital requirement. For this reason, our first solution for reducing the solvency capital requirement is to buy more of proportional reinsurance. While in the original sample portfolio there was proportional reinsurance for some lines of business (e.g. Motor, Aviation and Transport (50% cession) and Fire (30% cession)), now we introduce a proportional 50% cession for all lines of business, cf. Table 3.1.

Table 3.1: Proportional cessions

Line of business	A. Expected gross premium 2012	B. Proportional RI cession	C = A * B Ceded Premium 2012 (prop. RI)	D = A - C Retained Premium 2012 (prop. RI)
Motor vehicle liability insurance	20,000	50%	10,000	10,000
Other motor insurance	5,000	50%	2,500	2,500
Marine, aviation and transport insurance	5,000	50%	2,500	2,500
Fire and other damage to property insurance	15,000	50%	7,500	7,500
General liability insurance	5,000	50%	2,500	2,500
Total	50,000	50%	25,000	25,000

As far as non-proportional reinsurance is concerned, we first decided not to introduce any material changes. We still have to adjust the required non-proportional capacities to the new values of PMLs retained under proportional reinsurance. Table 3.2 provides the summary of the non-proportional reinsurance.

Table 3.2: Non-proportional reinsurance (Scenario 1)

Line of business	E. PML gross	F = E * (1 - B) PML net retained after prop. RI	G. Non-prop XL RI 2012	H. Non-prop XL RI Rate on GNPI	I = D * H XL RI premium 2012
Motor vehicle liability insurance	10,000	5,000	4,500 xs 500	1.0%	102
Other motor insurance	10,000	5,000	4,500 xs 500	2.5%	62
Marine, aviation and transport insurance	50,000	25,000	24,500 xs 500	9.6%	240
Fire and other damage to property insurance					0
Fire non natural catastrophe per risk	30,000	15,000	14,500 xs 500	11.4%	852
Fire natural catastrophe per event (EQ)	100,000	50,000	49,500 xs 500	5.9%	439
General liability insurance	5,000	2,500	2,000 xs 500	2.1%	52
Total					1,746

The reinsurance premium rates in column H were calculated in the same way as for the original contracts, cf. Table 2.3.1, column C and Annex A4. The rate for the Marine XL is the same as in the original portfolio because the protected exposure has not changed. For all other XL contracts, the rates have dropped because the exposure to the layers has decreased due to the higher proportional cession of 50%.

Table 3.3 provides the solvency capital requirement which results from the above reinsurance structure defined by Table 3.1 and Table 3.2.

Table 3.3: Total capital requirement for non-life underwriting risk and credit default risk

	SCR
A. NL underwriting risk	7,410
B. Default	2,959
C. Diversification effect	1,118
E = C - D: Total SCR	9,252

As we see, we have achieved a substantial reduction of solvency capital requirement compared to the original portfolio (Table 2.3.13), however the level of Solvency I (€ 7.8m) has not yet been achieved.

Now, we will adjust the non-proportional reinsurance and see what effect on the capital requirement such an adjustment will have. We will reduce the retentions of non-proportional contract to a € 250k. The following Table 3.4 summarizes the new non-proportional reinsurance portfolio.

Table 3.4: Non-proportional reinsurance (Variant 2)

Line of business	E. PML gross	F = E * (1 - B) PML net retained after prop. RI	G. Non-prop XL RI 2012	H. Non-prop XL RI Rate on Premium	I = D * H XL RI premium 2012
Motor vehicle liability insurance	10,000	5,000	4,750 xs 250	2.3%	230
Other motor insurance	10,000	5,000	4,750 xs 250	4.9%	124
Marine, aviation and transport insurance	50,000	25,000	24,750 xs 250	16.4%	410
Fire and other damage to property insurance					
Fire non natural catastrophe per risk	30,000	15,000	14,750 xs 250	18.9%	1,417
Fire natural catastrophe per event (EQ)	100,000	50,000	49,750 xs 250	8.1%	609
General liability insurance	5,000	2,500	2,250 xs 250	4.4%	111
Total					2,901

Pricing rates in column H were calculated in the same way as for the original contracts, cf. Table 2.3.1, column C and Table 3.2, column H and Annex A4.

With these adjusted non-proportional reinsurance contracts, the resulting capital requirement is as per Table 3.5.

Table 3.5: Total capital requirement for non-life underwriting risk and credit default risk (Scenario 2)

	SCR
A. NL underwriting risk	6,970
B. Default	2,989
C. Diversification effect	1,107
E = C - D: Total SCR	8,851

One can see that although the resulting capital requirement is getting closer to the one of Solvency I, it is still above it by about € 1.1m. The comparison of Table 3.3 and Table 3.5 shows that the capital requirement for the underwriting risk has dropped while the capital requirement for the counterparty default risk has increased. Both movements can be explained by the more extensive XL reinsurance covers in Scenario 2 - the net retained

underwriting risk exposure has dropped and the exposure to the credit default risk has increased due to the higher expected loss transferred to reinsurance.

As the next instrument for reducing the capital requirement, we now will examine the effect of improving the reinsurers' credit quality i.e. their credit ratings. In the previous calculations, we assumed that all reinsurance companies in the panel have an A rating. Now we change this assumption and assume that all reinsurance companies have the rating of an AA. Table 3.6 provides the parameters of non-proportional portfolio with the AA rating.

Table 3.6: Non-proportional reinsurance (Scenario 2, rating AA)

Line of business	E. PML gross	F = E * (1 - B) PML net retained after prop. RI	G. Non-prop XL RI 2012	H. Non-prop XL RI Rate on Premium	I = D * H XL RI premium 2012
Motor vehicle liability insurance	10,000	3,000	4,750 xs 250	2.5%	253
Other motor insurance	10,000	3,000	4,750 xs 250	5.4%	136
Marine, aviation and transport insurance	50,000	25,000	24,750 xs 250	18.1%	452
Fire and other damage to property insurance					
Fire non natural catastrophe per risk	30,000	15,000	14,750 xs 250	20.8%	1,559
Fire natural catastrophe per event (EQ)	100,000	50,000	49,750 xs 250	8.9%	670
General liability insurance	5,000	2,500	2,250 xs 250	4.9%	122
Total					3,191

While the reinsurance premium for proportional reinsurance does not depend on the rating, the premium for non-proportional reinsurance does. We assumed that a one notch upgrade in rating, i.e. a change from A to AA, costs 10% more in additional reinsurance premium. Pricing rates in Table 3.6, column H we obtained by applying a 10% loading on reinsurance pricing rates listed in Table 3.4, column H.

The resulting capital requirement is provided in Table 3.7.

Table 3.7: Total capital requirement for non-life underwriting risk and credit default risk (Option 2, rating AA)

	SCR
A. NL underwriting risk	6,968
B. Default	1,449
C. Diversification effect	623
E = C - D: Total SCR	7,794

The resulting capital requirement is now nearly equal to the one of Solvency I, i.e. the desired effect has been achieved. The capital requirement relief is equal to € 11.0m – 7.8m = 3.2m.

A comparison of Table 3.5 and Table 3.7 shows that the capital requirement for the underwriting risk has not changed much whereas the capital requirement for the counterparty credit default risk has dropped substantially. Such a substantial positive effect on the capital requirement is a result of improved credit quality of the involved reinsurance companies.

Now we compare the costs of reinsurance for the original reinsurance portfolio (cf. Table 2.1.2 and Table 2.3.1) and the updated reinsurance portfolio (Scenario 2, rating AA) which

under Solvency II leads to the same capital requirement as that of the original portfolio under Solvency I (cf. Table 3.1 and 3.4). Table 3.8 summarizes this comparison.

Table 3.8: Reinsurance premiums, original and updated reinsurance portfolio

Line of business	A. Reinsurance premium, prop RI, original portf.	B. Reinsurance premium, XL RI, original portf.	C = A + B Total reinsurance premium, original portf.	D. Reinsurance premium, prop RI, updated portf.	E. Reinsurance premium, XL RI, updated portf.	F = D + E Total reinsurance premium, updated portf.
Motor vehicle liability insurance	0	460	460	10,000	253	10,253
Other motor insurance	0	247	247	2,500	136	2,636
Marine, aviation and transport insurance	2,500	240	2,740	2,500	452	2,952
Fire and other damage to property insurance	4,500		4,500	7,500	0	7,500
Fire non natural catastrophe per risk		1,530	1,530		1,559	1,559
Fire natural catastrophe per event (EQ)		722	722		670	670
General liability insurance	0	222	222	2,500	122	2,622
Total	7,000	3,421	10,421	25,000	3,191	28,191

As can be seen from the last row of columns C and F in Table 3.8, the total reinsurance premium of the updated portfolio is about € 18 million higher than that of the original portfolio. The achieved relief of € 3.2 million in the capital requirement has been achieved at the cost of € €28.2m – €10.4m = €17.8 million of additional reinsurance premium! At first glance, it may appear that seeking capital relief through reinsurance, which costs over 500% more than the capital relief itself, makes no economic sense! However, one can easily observe that in the case of proportional reinsurance, the reinsurance premium does not reflect well the actual capital cost of reinsurance protection. It can be shown that if the expected loss ratio materializes, the real cost (we called it P & L statement costs) of proportional reinsurance can be calculated as shown below.

“P & L Statement Costs of prop RI” =

“Reinsurance Premium” * (1 – “Expected LR” – “RI Commission [%]”).

Table 3.9 summarizes the P & L statement costs of proportional reinsurance in the original and updated portfolio.

Table 3.9: P & L statement costs of proportional reinsurance, original and updated portfolio

Line of business	G. [%] Expected ultimate loss ratio	H. Reinsurance commission original portf.	I. Reinsurance commission updated portf.	J = A * (1 - G - H) P & L statement costs of prop RI original portf.	K = D * (1 - G - I) P & L statement costs of prop RI updated portf.
Motor vehicle liability insurance	70%	nil.	18%	0	1,200
Other motor insurance	60%	mil.	27%	0	325
Marine, aviation and transport insurance	80%	10%	9%	250	275
Fire and other damage to property insurance	70%	20%	18%	450	900
General liability insurance	60%	nil.	27%	0	325
Total				700	3,025

In columns H and I of Table 3.9, we present reinsurance commissions (flat) for different lines of business which were calculated based on the assumed 10% profit margin for the reinsurance company⁷ when it has an A rating (original reinsurance, column H) and a 10% lower commission when it has an AA rating (updated reinsurance structure, column I).

⁷ Reinsurance company Profit Margin as percentage of the premium = 100% reinsurance premium minus ceded loss as [%] of reinsurance premium minus reinsurance commission as [%] of reinsurance premium.

Now we can compare the overall costs of the original and updated reinsurance portfolio made of reinsurance premium for non-proportional XL reinsurance and P & L statement costs of proportional reinsurance, cf. Table 3.10.

Table 3.10: total P & L statement costs of reinsurance, original and updated portfolio

Line of business	L = J + B total P & L statement costs of RI, original portf.	M = K + E total P & L statement costs of RI, updated portf.
Motor vehicle liability insurance	460	1,453
Other motor insurance	247	461
Marine, aviation and transport insurance	490	727
Fire and other damage to property insurance	2,702	3,129
General liability insurance	222	447
Total	4,121	6,216

Based on the P & L statement costs of reinsurance, the additional costs for the updated reinsurance structure calculates to € 6.2m – 4.1m = 2.1m – the amount which no longer exceeds the achieved € 3.2 million relief in capital requirement.

To evaluate the efficiency of reinsurance as the solvency capital relief instrument, we now introduce the *relative costs of capital ratio* which is calculated as the cost of reinsurance divided by the achieved capital relief. Obviously, the lower this ratio, the better the efficiency of the given reinsurance structure as the capital relief instrument. Furthermore, the straight forward cost of capital approach afforded by this ratio also allows a direct comparison of reinsurance as a capital relief instrument with capital borrowed from the capital markets. That is if the rates offered by the capital markets are higher than the relative cost of capital ratio resulting for a prospected reinsurance structure, then a given reinsurance structure has a cost advantage over debt instruments as a solvency capital relief instrument.

In our case, we calculate the relative cost of capital $2.1\text{m} / 3.2\text{m} = 65.6\%$ (cost of capital relief divided by the capital relief) which clearly is very high. Most likely, the insurance company would be able to borrow from the capital markets at a much lower rate.

To increase the attractiveness of reinsurance as a solvency capital relief instrument compared to borrowing, the cost of reinsurance protection should be reduced substantially. One conventional way to reduce the costs of reinsurance protection is to replace conventional proportional contracts with flat reinsurance commissions with the so called *financial quota share* (FQS) contracts – a type of proportional contracts which usually has a broad sliding scale reinsurance commission that depends on the underwriting performance of the contract. Such contracts are called *financial* because along with the transfer of insurance risk they are usually motivated also by financing objectives such as achieving a solvency capital relief.

The diagram below provides an example of a sliding scale reinsurance commission. In the loss ratio range of 45%-85% the reinsurance commission is chosen in the way that the total of the loss ratio and commission is kept constant at 95%. Obviously, the resulting reinsurer's margin in this broad loss ratio range is 5%.

The cost of reinsurance for a financial quota share contract is equal to the reinsurer's margin, in the case of the above diagram 5% of the reinsurance premium. We would like to comment that typically a 5% margin is rather high, but we chose it at 5% to be conservative; in the international reinsurance markets the margins of 2-3% are quite common.

We now calculate the costs of reinsurance for the updated portfolio when the conventional quota share contracts have been replaced with a financial quota share contract for the full amount of proportional reinsurance, with the reinsurer's margin of 5%. The financial quota share contracts do not necessarily need to be taken out for the whole account, as the per-line-of-business contracts are possible and can be often seen in practice.

Table 3.11 summarizes the P & L statement costs of proportional reinsurance in the original and updated portfolio.

Table 3.11: P & L statement costs of proportional reinsurance, original and updated portfolio (with FQS)

Line of business	G. [%] Expected ultimate loss ratio	H. Reinsurance commission original portf.	I. Reinsurance commission updated portf.	J = A * (1 - G - H) P & L statement costs of prop RI original portf.	K = Total D * 5% P & L statement costs of prop RI updated portf.
Motor vehicle liability insurance	70%	nil.	Whole account sliding scale with reinsurer's margin of 5%	0	
Other motor insurance	60%	nil.		0	
Marine, aviation and transport insurance	80%	10%		250	
Fire and other damage to property insurance	70%	20%		450	
General liability insurance	60%	nil.		0	
Total				700	1,250

We can now compare the overall costs of the original and updated reinsurance portfolio made of reinsurance premium for non-proportional XL reinsurance and P & L statement costs of proportional reinsurance, cf. Table 3.12.

Table 3.12: total P & L statement costs of reinsurance, original and updated portfolio (with FQS)

Line of business	L = J + B total P & L statement costs of RI, original portf.	M = K + E total P & L statement costs of RI, updated portf.
Total	4,121	4,441

Now, the additional reinsurance costs for the updated reinsurance structure amount to only € 4.4m – 4.1m = 0.3m. With the achieved capital requirement relief of € 7.8m we calculate the relative cost of capital as $0.3/3.2 = 9.4\%$. Hence, by introducing the financial quota share reinsurance we have achieved a substantial reduction in the cost of capital. Most likely, the resulting cost of capital will compare favorably with the borrowing costs for an emerging market mid-size insurance company. Reinsurance has proven to be a competitive capital relief instrument!

We must point out however that to make good use of financial quota share contracts for achieving cost efficient solvency relief, one needs to ensure that such contracts must fulfill certain requirements to qualify as reinsurance for regulatory purposes and hence achieve the desired solvency relief effect. In many cases, the ceding company will be required by law to prove to the market regulator that the contract transfers a substantial amount of risk and hence can be considered a reinsurance contract. If this cannot be proved, the contract will be considered not as reinsurance but as a purely financial instrument and therefore no solvency capital relief effect will be granted. Please refer to the paper *“Insurance Risk Transfer and Categorization of Reinsurance Contracts”* by the same authors.

4. Conclusions

In this paper, we have compared solvency capital requirements under Solvency I and Solvency II for a sample mid-size insurance portfolio. We have shown that the capital requirement under the Solvency II regime is substantially higher than under the Solvency I regime. Hence, according to our result, changing the solvency capital regime from Solvency I to Solvency II will lead to a substantial additional solvency capital requirement which might represent a heavy burden for the company’s shareholders.

One way to reduce the capital requirement under Solvency II is to increase reinsurance protection, which will reduce the net retained risk exposure and hence also the solvency capital requirement. Therefore, we proposed an extended reinsurance structure which under Solvency II brings the capital requirement back to the level of that required under Solvency I. In a step-by-step approach we demonstrate the extent of solvency relief attained by the insurer by applying different possible adjustments in the reinsurance structure: increasing the cession to proportional reinsurance, reducing the retention of non-proportional reinsurance, and selecting better rated reinsurers for its reinsurance program.

To evaluate the efficiency of reinsurance as the solvency capital relief instrument, we have introduced a cost-of-capital based approach which puts the achieved capital relief in relation to the costs of extending the reinsurance protection. This approach allows a direct comparison of reinsurance as capital relief instrument with debt instruments available in the capital market. With the help of the introduced approach, we have shown that the best capital relief efficiency under all examined reinsurance alternatives is achieved when a financial quota share contract is chosen for proportional reinsurance. For the sample insurance portfolio and the proposed reinsurance structure, we achieved the cost of capital required for the capital relief of less than 10% - a highly competitive rate in comparison with the cost of corporate borrowing in the capital market.

Annex A1

Solvency Capital Requirements according to Solvency I

In order to calculate Solvency Capital Requirements, first one needs to calculate the Retention Ratio

$$\text{Retention Ratio} = \left(\begin{array}{l} + \text{retained loss report yr.} \\ + \text{retained loss 1}^{\text{st}} \text{ yr. before report yr.} \\ + \text{retained loss 2}^{\text{nd}} \text{ yr. before report yr.} \end{array} \right) / \left(\begin{array}{l} + \text{gross loss report yr.} \\ + \text{gross loss 1}^{\text{st}} \text{ yr. before report yr.} \\ + \text{gross loss 2}^{\text{nd}} \text{ yr. before report yr.} \end{array} \right).$$

Then the Premium Index and the Claims Index can be calculated. The Premium Index is calculated as follows

Premium Index =

$$\begin{aligned} & \text{Retention Ratio} * (+ 0.18 * \min (\text{adjusted gross premium for report yr., } \text{€ } 57.5 \text{ M}) \\ & \quad + 0.16 * \max (\text{adjusted gross premium for report yr.} - \text{€ } 57.5 \text{ M, } 0)), \end{aligned}$$

The expression in brackets in the above formula for the Premium Index is equal to the adjusted gross premium for the report year up to the limit of € 57.4 M multiplied by 0.18 plus the premium in excess of € 57.5 M, if any, multiplied by 0.16. Adjusted gross premium is equal to the gross premium plus 50% of the premium from the lines Aviation Liability, Water Transport Liability and General Liability (if any).

Claims Index is calculated as follows

Claims Index =

$$\begin{aligned} & \text{Retention Ratio} * (+ 0.26 * \min (\text{mean gross loss of last three yrs., } \text{€ } 40.3 \text{ M}) \\ & \quad + 0.23 * \max (\text{mean gross loss of last three yrs.} - \text{€ } 40.3 \text{ M, } 0)), \end{aligned}$$

where mean gross loss of last three years is calculated as

$$\begin{aligned} \text{mean gross loss of last three yrs} = & \\ & (+ \text{adjusted gross loss report yr.} \\ & + \text{adjusted gross loss 1}^{\text{st}} \text{ yr. before report yr.} \\ & + \text{adjusted gross loss 2}^{\text{nd}} \text{ yr. before report yr.}) / 3 \end{aligned}$$

The expression in brackets in the above formula for the Claims Index is equal to the mean gross loss of last three years up to the limit of € 40.3 M multiplied by 0.26 plus the mean gross loss of last three years in excess of € 40.3 M if any multiplied by 0.23. Adjusted gross loss is equal to the gross loss plus 50% of the gross loss from the lines Aviation Liability, Water Transport Liability and General Liability (if any).

Finally, the Solvency Capital Requirement is calculated as maximum of both Premium Index and Claims Index

$$\text{Solvency Capital Requirement} = \max(\text{Premium Index}, \text{Claims Index})$$

If the resulting Solvency Capital Requirement for the report year is lower than for the previous year, the following further calculation needs to be carried out.

Loss reserve ratio =

$$\max(1, \text{loss reserves at the end of report yr.} / \text{loss reserves at the beginning of report yr.})$$

$$\text{Solvency Capital Requirement} = \max(\max(\text{Premium Index}, \text{Claims Index}), \\ \text{Loss reserve ratio} * \text{Solvency Capital Requirement previous yr.})$$

Annex A2

Solvency Capital Requirements for non-life underwriting risk under Solvency II (QIS5) standard formula

This Annex describes the calculation of solvency capital requirements for non-life underwriting risk according to Solvency II (QIS5) standard formula, as defined by EU document “QIS5 Technical Specification” (Brussels, July 2010).

The Annex is organized as follows. Section A2.1 describes the overall approach for calculating the solvency capital requirement for non-life underwriting risk based on the results of three sub-modules: non-life premium and reserve risk, non-life lapse risk and non-life catastrophe risk sub-module. Sections A2.2 and A2.4 then provide a detailed explanation of the capital requirement calculation for the sub-modules non-life premium and reserve risk (Section A2.3) and non-life catastrophe risk (Section A2.4). Section A2.3 provides an explanation on the objective and motivation behind the non-life lapse risk sub-module. However, the section provides no calculation details as we assume this risk category does not materialize for the sample portfolio under consideration. Finally Section A2.4 explains the aggregation of the capital requirements for the non-life underwriting risk module and the counterparty default risk module (cf. Annex 3). This aggregation takes into account some correlation assumptions between these two risk categories.

A2.1 Calculation of the overall capital requirement for non-life underwriting risk

The non-life underwriting risk module consists of the following sub-modules:

- The non-life premium and reserve risk sub-module
- The non-life lapse risk sub-module
- The non-life catastrophe risk sub-module

Capital requirements for each module are denoted by

NL_{pr} = Capital requirement for non-life premium and reserve risk

NL_{lapse} = Capital requirement for non-life lapse risk

NL_{CAT} = Capital requirement for non-life catastrophe risk

The resulting solvency Capital requirement for non-life underwriting risk is denoted by

SCR_{nl} = Capital requirement for non-life underwriting risk

It calculates by combining the capital requirements for the non-life sub-risks using a correlation matrix as follows

$$SCR_{nl} = \sqrt{\sum CorrnL_{r,c} \cdot NL_r \cdot NL_c}$$

where

$CorrnL_{r,c}$ = The entries of the correlation matrix $CorrnL$

$NL_r, NL_c =$ Capital requirement for individual non-life underwriting sub-risks according to the rows and columns of correlation matrix $CorrNL$

and where the correlation matrix $CorrNL$ is defined by

$CorrNL$	NL_{pr}	NL_{lapse}	NL_{CAT}
NL_{pr}	1		
NL_{lapse}	0	1	
NL_{CAT}	0.25	0	1

With this correlation matrix, the overall solvency capital requirement for non-life underwriting risk calculates to

$$SCR_{nl} = \sqrt{NL_{pr}^2 + NL_{lapse}^2 + NL_{CAT}^2 + 0.25NL_{pr} NL_{CAT}}$$

A2.2 Calculation of the capital requirement for non-life premium and reserve risk

This module considers two main sources of underwriting risk, premium risk and reserve risk. Premium risk results from fluctuations in the timing, frequency and severity of insured events. Premium risk entails the risk that premium provisions turn out to be insufficient to cover claims in full or need to be increased. Reserve risk results from fluctuations in the timing and amount of claim settlements.

In order to carry out the non-life premium and reserve risk calculation, the following parameters need to be determined

$PCO_{lob} =$	Provisions for claims outstanding (best estimate) for each LoB net of recoverables from reinsurance and special purpose vehicles
$P_{lob}^{t,written} =$	Estimate for net written premium for each LoB during the forthcoming year
$P_{lob}^{t,earned} =$	Estimate for net earned premium for each LoB during the forthcoming year
$P_{lob}^{t-1,written} =$	Net written premium for each LoB during the previous year
$P_{lob}^{PP} =$	Present value of net premiums of existing contracts which are expected to be earned after the following year for each LoB

The term P_{lob}^{PP} is only relevant for contracts with a coverage period that exceeds the following year. For annual contracts without renewal options P_{lob}^{PP} is zero. Undertakings may not calculate P_{lob}^{PP} where it is likely not to be material compared to $P_{lob}^{t,earned}$.

The capital requirement for premium and reserve risk is determined as follows:

$$NL_{pr} = \rho(\sigma) \cdot V$$

where

V = Volume measure
 σ = Combined standard deviation
 $\rho(\sigma)$ = A function of the combined standard deviation

The function $\rho(\sigma)$ is specified as follows:

$$\rho(\sigma) = \frac{\exp(N_{0.995} \cdot \sqrt{\log(\sigma^2 + 1)})}{\sqrt{\sigma^2 + 1}} - 1$$

where

$N_{0.995}$ = 99.5% quantile of the standard normal distribution (≈ 2.576)

The function $\rho(\sigma)$ is set in such a way that, assuming a lognormal distribution of the underlying risk, a risk capital requirement consistent with the VaR 99.5% calibration objective is produced. Roughly, $\rho(\sigma) \approx 3 \cdot \sigma$.

The volume measure V and the combined standard deviation σ for the overall non-life insurance portfolio are determined in two steps as follows:

- For each individual LoB, the standard deviations and volume measures for both premium risk and reserve risk are determined;
- The standard deviations and volume measures for the premium risk and the reserve risk in the individual LoBs are aggregated to derive an overall volume measure V and a combined standard deviation σ .

The calculations needed to perform these two steps are set out below.

Step 1: Volume measures and standard deviations per LoB

The following numbering of LoBs applies for the calculation:

Number	Line of business
1	Motor vehicle liability
2	Motor, other classes
3	Marine, aviation, transport (MAT)
4	Fire and other property damage
5	Third-party liability
6	Credit and suretyship
7	Legal expenses
8	Assistance
9	Miscellaneous
10	Non-proportional reinsurance – property
11	Non-proportional reinsurance – casualty
12	Non-proportional reinsurance – MAT

For each LoB, the volume measures and standard deviations for premium and reserve risk are denoted as follows:

$V_{(prem,lob)} =$ The volume measure for premium risk

$V_{(res,lob)} =$ The volume measure for reserve risk

$\sigma_{(prem,lob)} =$ Standard deviation for premium risk

$\sigma_{(res,lob)} =$ Standard deviation for reserve risk

The volume measure for premium risk in the individual LoB is determined as follows:

$$V_{(prem,lob)} = \max(P_{lob}^{t,written}; P_{lob}^{t,earned}; P_{lob}^{t-1,written}) + P_{lob}^{PP}$$

If the undertaking has committed to its regulator that it will restrict premiums written over the period so that the actual premiums written (or earned) over the period will not exceed its estimated volumes, the volume measure is determined only with respect to estimated premium volumes, so that in this case:

$$V_{(prem,lob)} = \max(P_{lob}^{t,written}; P_{lob}^{t,earned}) + P_{lob}^{PP}$$

The market-wide estimates of the net standard deviation for premium risk for each line of business are:

<i>LoB</i>	<i>standard deviation for premium risk (net of reinsurance)</i>
<i>Motor vehicle liability</i>	10%· NP_{lob}
<i>Other motor</i>	7%· NP_{lob}
<i>MAT</i>	17%· NP_{lob}
<i>Fire</i>	10%· NP_{lob}
<i>3rd-party liability</i>	15%· NP_{lob}
<i>Credit</i>	21.5%· NP_{lob}
<i>Legal expenses</i>	6.5%· NP_{lob}
<i>Assistance</i>	5%· NP_{lob}
<i>Miscellaneous</i>	13%· NP_{lob}
<i>Np reins (prop)</i>	17.5%
<i>Np reins (cas)</i>	17%
<i>Np reins (MAT)</i>	16%

The adjustment factor for non-proportional reinsurance NP_{lob} of a line of business allows undertakings to take into account the risk-mitigating effect of particular per risk excess of loss reinsurance. Undertakings may choose for each line of business to set the adjustment factor to 1 or to calculate it according to the following algorithm.

Box: Calculation of NP_{lob} factors

The adjustment factor for non-proportional reinsurance should only be calculated in relation to per risk excess of loss reinsurance which complies with the following conditions:

- it covers all insurance claims that the insurance or reinsurance undertaking may incur in the segment during the following year;
- it allows for reinstatement;

- it meets the requirements for risk mitigation techniques, set out in subsection SCR.13 of the defined by EU document “QIS5 Technical Specification” (Brussels, July 2010).

$$NP_{lob} = \sqrt{\frac{1 + (\Omega_{lob}^{net} / M_{lob}^{net})^2}{1 + (\Omega_{lob}^{gross} / M_{lob}^{gross})^2}}$$

where

$$M_{lob}^{net} = M_{lob}^{gross} \cdot [1 - F_{m+\sigma^2, \sigma}(a+b) + F_{m+\sigma^2, \sigma}(a)] + a \cdot [F_{m, \sigma}(a+b) - F_{m, \sigma}(a)] - b \cdot [1 - F_{m, \sigma}(a+b)]$$

$$\Omega_{lob}^{net} = \left(\left((\Omega_{lob}^{gross})^2 + (M_{lob}^{gross})^2 \right) \cdot [1 - F_{m+2\sigma^2, \sigma}(a+b) + F_{m+2\sigma^2, \sigma}(a)] + a^2 \cdot [F_{m, \sigma}(a+b) - F_{m, \sigma}(a)] - 2b \cdot M_{lob}^{gross} \cdot [1 - F_{m+\sigma^2, \sigma}(a+b)] + b^2 \cdot [1 - F_{m, \sigma}(a+b)] - (M_{lob}^{net})^2 \right)^{1/2}$$

$$\sigma = \sqrt{\ln \left(1 + \left(\frac{\Omega_{lob}^{gross}}{M_{lob}^{gross}} \right)^2 \right)}$$

$$m = \ln M_{lob}^{gross} - \frac{\sigma^2}{2}$$

$$M_{lob}^{gross} = \begin{cases} \tilde{M}_{lob}^{gross} & \text{if } S \geq 1 \\ S \cdot \tilde{M}_{lob}^{gross} & \text{otherwise} \end{cases}$$

$$\Omega_{lob}^{gross} = \begin{cases} \tilde{\Omega}_{lob}^{gross} & \text{if } S \geq 1 \\ S \cdot \tilde{\Omega}_{lob}^{gross} & \text{otherwise} \end{cases}$$

$$S = \sqrt{\frac{n \cdot \sigma_{(prem, gross, lob)}^2 \cdot V_{(prem, gross, lob)}^2}{N \cdot \left((\tilde{\Omega}_{lob}^{gross})^2 + (\tilde{M}_{lob}^{gross})^2 \right)}}$$

The terms used in these formulas are defined as follows

$\tilde{M}_{lob}^{gross} =$	Average cost per claim gross of reinsurance per LoB estimated from the claims of the last n years where $n \geq 1$
$\tilde{\Omega}_{lob}^{gross} =$	Standard deviation of the cost per claim gross of reinsurance per LoB estimated with the standard estimator from the claims of the last n years where $n \geq 1$
$a =$	Retention of non-proportional reinsurance contract
$b =$	Limit of non-proportional reinsurance contract
$F_{m, \sigma} =$	Distribution function of a Lognormal random variable with parameters (m, σ)
$F_{m+\sigma^2, \sigma} =$	Distribution function of a Lognormal random variable with parameters $(m + \sigma^2, \sigma)$
$F_{m+2\sigma^2, \sigma} =$	Distribution function of a Lognormal random variable with parameters $(m + 2\sigma^2, \sigma)$
$n =$	Number of years used in the estimation of $\tilde{M}_{lob}^{gross}$ and $\tilde{\Omega}_{lob}^{gross}$
$N =$	Number of claims during the last n years

$\sigma_{(prem,gross,lob)} =$ Standard deviation for premium risk gross of reinsurance, calculated by putting the adjustment factor NP_{lob} to 1.

$V_{(prem,gross,lob)} =$ Volume measure for premium risk gross of reinsurance, calculated in the same way as the usual volume measure but based on gross premiums instead of net premiums

Where the excess of loss reinsurance contract has no limit the adjustment factor for non-proportional reinsurance of a line of business shall be calculated in the same way as set out above, but with the following changes:

$$M_{lob}^{net} = M_{lob}^{gross} \cdot F_{m+\sigma^2, \sigma}(a) + a \cdot [1 - F_{m, \sigma}(a)]$$

$$\Omega_{lob}^{net} = \left(\left((\Omega_{lob}^{gross})^2 + (M_{lob}^{gross})^2 \right) \cdot F_{m+2\sigma^2, \sigma} + a^2 \cdot [1 - F_{m, \sigma}(a)] - (M_{lob}^{net})^2 \right)^{1/2}$$

End Box

The volume measure for reserve risk for each individual LoB is determined as follows:

$$V_{(res,lob)} = PCO_{lob}$$

The market-wide estimates of the net of reinsurance standard deviation for reserve risk for each line of business are:

LoB_t	<i>standard deviation for reserve risk (net of reinsurance)</i>
<i>Motor vehicle liability</i>	9.5%
<i>Other motor</i>	10%
<i>MAT</i>	14%
<i>Fire</i>	11%
<i>3rd-party liability</i>	11%
<i>Credit</i>	19%
<i>Legal expenses</i>	9%
<i>Assistance</i>	11%
<i>Miscellaneous</i>	15%
<i>Np reins (prop)</i>	20%
<i>Np reins (cas)</i>	20%
<i>Np reins (MAT)</i>	20%

The standard deviation for premium and reserve risk in the individual LoB is defined by aggregating the standard deviations for both sub-risks under the assumption of a correlation coefficient of $\alpha = 0.5$:

$$\sigma_{lob} = \frac{\sqrt{(\sigma_{(prem,lob)} V_{(prem,lob)})^2 + 2\alpha \sigma_{(prem,lob)} \sigma_{(res,lob)} V_{(prem,lob)} V_{(res,lob)} + (\sigma_{(res,lob)} V_{(res,lob)})^2}}{V_{(prem,lob)} + V_{(res,lob)}}$$

The overall volume measure for each LoB, V_{lob} is obtained as follows:

$$V_{lob} = (V_{(prem,lob)} + V_{(res,lob)}) \cdot (0.75 + 0.25 \cdot DIV_{lob})$$

where

$$DIV_{lob} = \frac{\sum_j (V_{(prem,j,lob)} + V_{(res,j,lob)})^2}{(\sum_j (V_{(prem,j,lob)} + V_{(res,j,lob)}))^2}$$

and where the index j denotes the geographical segments as set out in Annex M to the Technical Specification and $V_{(prem,j,lob)}$ and $V_{(res,j,lob)}$ denote the volume measures as defined above but restricted to the geographical segment j .

However, the factor DIV_{lob} should be set to 1 for the line of business credit and suretyship and where the standard deviation for premium or reserve risk of the line of business is an undertaking-specific parameter.

For an undertaking offering insurance in one geographic segment only, the factor is equal to 1.

Undertakings active in more than one geographic segment may choose to allocate all of their business in a line of business to the main geographical segment in order to simplify the calculation.

Step 2: Overall volume measures and standard deviations

The overall volume measure V for all LoBs is determined as the total of the volume measures for each LoB:

$$V = \sum_{all\ LoBs} V_{lob}$$

The overall standard deviation σ for all LoBs is determined as follows:

$$\sigma = \sqrt{\frac{1}{V^2} \cdot \sum_{r,c} CorrLoB_{r,c} \cdot \sigma_r \cdot \sigma_c \cdot V_r \cdot V_c}$$

where

$CorrLoB_{r,c}$ = The entries of the correlation matrix $CorrLoB$

σ_r, σ_c = Standard deviations for individual line of business according to the rows and columns of correlation matrix $CorrLoB$

V_r, V_c = Volume measures for individual line of business according to the rows and columns of correlation matrix $CorrLoB$

The correlation matrix *CorrLoB* is defined as follows

<i>CorrLoB</i>	1	2	3	4	5	6	7	8	9	10	11	12
1: Motor vehicle liability	1											
2: Other motor	0,5	1										
3: MAT	0,5	0,25	1									
4: Fire	0,25	0,25	0,25	1								
5: 3rd party liability	0,5	0,25	0,25	0,25	1							
6: Credit	0,25	0,25	0,25	0,25	0,5	1						
7: Legal exp.	0,5	0,5	0,25	0,25	0,5	0,5	1					
8: Assistance	0,25	0,5	0,5	0,5	0,25	0,25	0,25	1				
9: Miscellaneous.	0,5	0,5	0,5	0,5	0,5	0,5	0,5	0,5	1			
10: Np reins. (property)	0,25	0,25	0,25	0,5	0,25	0,25	0,25	0,5	0,25	1		
11: Np reins. (casualty)	0,25	0,25	0,25	0,25	0,5	0,5	0,5	0,25	0,25	0,25	1	
12: Np reins. (MAT)	0,25	0,25	0,5	0,5	0,25	0,25	0,25	0,25	0,5	0,25	0,25	1

A2.3 Non-life lapse risk

Non-life insurance contracts can include policyholder options which significantly influence the obligations arising from them. Examples for such options are options to terminate a contract before the end of the previously agreed insurance period and options to renew contracts according to previously agreed conditions. Where such policyholder options are included in a non-life insurance contract, the calculation of premium provisions is based on assumptions about the exercise rates of these options. Lapse risk is the risk that these assumptions turn out to be wrong or need to be changed.

Where non-life insurance contracts do not include policyholder options or where the assumptions about the exercise rate of such options have no material influence on premium provisions, the contracts do not need to be included in the calculations of the lapse risk sub-module. Where this is the case for the whole portfolio of an undertaking (except for a non-material part) the three components of the sub-module can be set to zero. In this Annex, we assume that this is the case and do not provide calculation details for the non-life lapse risk.

A2.4 Calculation of the capital requirement for non-life catastrophe risk

Under the non-life underwriting risk module, catastrophe risk is defined in the Solvency II Framework Directive (Directive 2009/138/EC) as: “the risk of loss, or of adverse change in the value of insurance liabilities, resulting from significant uncertainty of pricing and provisioning assumptions related to extreme or exceptional events.”

CAT risks stem from extreme or irregular events that are not sufficiently captured by the capital requirements for premium and reserve risk. The catastrophe risk capital requirement has to be calibrated at the 99.5% VaR (annual view).

The CAT risk sub-module under the standard formula should be calculated using one of the following alternative methods (or as a combination of both): Standardized Scenarios Method (method 1) or Factor Based Method (method 2) whereas the method 1 should be chosen for all exposures where possible. As usually this will be the case, in this Annex we do not provide any details on the method 2.

The CAT risk sub-module differentiates between natural catastrophes and man-made catastrophes. Perils considered under natural catastrophes are: Windstorm, Flood, Earthquake, Hail, Subsidence. Man-made catastrophes are extreme or exceptional events arising from Motor, Fire, Marine, Aviation, Liability, Credit & Suretyship, Terrorism.

The capital requirement for non-life catastrophe risk is calculated as follows:

$$NL_CAT = \sqrt{(NL_CAT_{Nat_cat})^2 + (NL_CAT_{Man_made})^2}$$

where

$NL_CAT_{Nat_cat}$ = Catastrophe capital requirement for natural catastrophes net of risk mitigation
 $NL_CAT_{Man_made}$ = Catastrophe capital requirement for man-made net of risk mitigation

In the following, we assume that the man-made catastrophe exposure is remote and do not provide any further details on this sub-module.

For calculating $NL_CAT_{Nat_cat}$, firstly natural catastrophe capital requirements at country level should be aggregated to estimate the capital requirement at peril level:

$$CAT_{peril} = \sqrt{\sum_{i,j} Corr_{ctry,i,j} \cdot CAT_{peril_ctry,net,i} \cdot CAT_{peril_ctry,net,j}}$$

where

CAT_{peril} = Catastrophe capital requirement for each peril type = Windstorm, Earthquake, Flood, Hail and Subsidence

$CAT_{peril_ctry,net,i,j}$ = Catastrophe capital requirement for each peril type by country. Where there are separate reinsurance programs for each country the aggregations (across countries) are done net of reinsurance.

$Corr_{ctry,i,j}$ = The entries of the correlation matrix between countries i, j.

Parameters of the correlation matrix are provided in the “Technical Specification”. As usually, insurance undertaking will be operating in one country only, we do not provide this lengthy correlation matrix in this Annex.

Then, capital requirements at peril level should be aggregated to estimate the catastrophe capital requirements at total level:

$$NL_CAT_{Nat_cat} = \sqrt{\sum_{i,j} Corr_{peril,i,j} \cdot CAT_{peril,net,i} \cdot CAT_{peril,net,j}}$$

where

$NL_CAT_{Nat_cat}$ = Catastrophe capital requirement for non-life net of risk mitigation

$CAT_{peril,net,i,j}$ = Catastrophe capital requirement for each peril. Where there are separate reinsurance programs for each country the aggregations (across countries) are done net of reinsurance.

$Corr_{peril,i,j}$ = The entries of the correlation matrix between perils i, j.

The peril correlation matrix $Corr_{peril}$ is as follows

	Windstorm	Earthquake	Flood	Hail	Subsidence
Windstorm	1.00				
Earthquake	0.00	1.00			
Flood	0.25	0.00	1.00		
Hail	0.25	0.00	0.00	1.00	
Subsidence	0.00	0.00	0.00	0.00	1.00

Notice that among all perils, some correlation is assumed for the pairs (windstorm; flood) and (windstorm; hail) only.

The capital requirements per peril, country CAT_{peril_ctry} gross of reinsurance are obtained from the sum insured aggregates per cresta zone, relativity factor for each zone and the zone aggregation matrix per country. Then the recoverables of reinsurance contract are taken into account. This provides the capital requirements per peril and country net of reinsurance $CAT_{peril_ctry,net}$.

For example, for earthquake, the calculation is as follows:

$$CAT_{EQ_ctry} = Q_{CTRY} \sqrt{\sum_{r,c} AGG_{r,c} \cdot WTIV_{ZONE,r} \cdot WTIV_{ZONE,c}}$$

where

$$WTIV_{ZONE} = F_{ZONE} \cdot TIV_{ZONE}$$

$$TIV_{ZONE} = TIV_{ZONE_Fire} + TIV_{ZONE_MAT}$$

TIV_{ZONE_Fire} = Total insured value for Fire and other damage by zone

TIV_{ZONE_MAT} = Total insured value for Marine by zone

Q_{CTRY} = 1 in 200 year factor for each country. These factors for different perils, countries are provided in Annex L5 to the “Technical Specification”

$AGG_{r,c}$ = Rows and columns of aggregation matrix by country. These values are provided in the excel spreadsheet “parameters for non-life catastrophe” available from EIOPA website (European Insurance and Occupational Pensions Authority) under the QIS5 helper tabs

F_{ZONE} = Relativity factors for each zone by country. These values are provided in the excel spreadsheet “parameters for non-life catastrophe” available from EIOPA website (European Insurance and Occupational Pensions Authority) under the QIS5 helper tabs

$WTIV_{ZONE}$ = Geographically weighted total insured value by zone

Finally through taking into account risk mitigating effect of reinsurance, the capital requirement for earthquake net of reinsurance

$$CAT_{EQ_ctry,net}$$

can be calculated.

Capital requirements for other perils can be calculated analogously, for more details consult “Technical Specification”.

Undertakings should note that netting down would not always be possible at a country level. For example you may have a European windstorm program in which case this would still be gross and not adjusted for risk mitigation until aggregating at country level. When netting down, undertakings should take care to adjust and interpret formulae accordingly.

A2.5 Aggregation of capital requirements for non-life underwriting risk and default risk

Aggregation of capital requirements for non-life underwriting risk and counterparty default risk takes into account some correlation assumptions. These assumptions are summarized by the following matrix (Technical Specification, SCR1.32)

	j	Non-life	Default
i			
Non-life		1	
Default		0.25	1

The aggregation calculates with the following

$$SCR_{nl+def} = \sqrt{\sum_{i,j} Corr_{i,j} \cdot SCR_{nl} \cdot SCR_{def}}$$

Calculation of the capital requirement for the counterparty default SCR_{def} risk is described in Annex A3.

Annex A3

Solvency capital requirements for reinsurance counterparty default risk under Solvency II (QIS5) standard formula

This specification follows “Technical Specification QIS5”, Section SCR.6 (SCR Counterparty risk module), pp. 193-203.

QIS5 differentiates between two kinds of counterparty risk exposure, denoted by type1 and type2 exposures. The class of type1 exposures covers the exposures which may not be diversified and where the counterparty is likely to be rated. Reinsurance arrangements belong to this type 1 class of exposures. The class of type2 exposures covers the exposures which are usually diversified and where the counterparty is likely to be unrated. Example of this class are receivables from intermediaries, policyholders debtors etc. Please refer to QIS5 Sections SCR6.2-6.4 for more details. We will consider only credit default risk exposure from reinsurance arrangements (type1) and assume there's no exposure of type 2.

Solvency capital requirements for the type 1 of exposure can be calculated as (SCR6.6):

$$SCR = \min(LGD_1 + LGD_2 + \dots + LGD_N; q \cdot \sqrt{V})$$

where

LGD_i = Loss-given-default for counterparty i

q = Quantile factor

V = Variance of the loss distribution for the credit risk exposure

According to SCR6.16, LGD_i can be calculated as

$$LGD_i = \max((1 - RR) \cdot (Recoverables_i + RM_i - Collateral_i); 0)$$

where

RR = Recovery rate for reinsurance arrangements. According to SCR6.52, for reinsurance arrangements, $RR = 50\%$ can be assumed.

$Recoverables_i$ = Best estimate recoverables from the reinsurance contract i. We assume that $Recoverables_i$ = expected loss ceded to the reinsurance contract i plus reinsurance share on provisions for claims provisions.

$Collateral_i$ = Risk-adjusted value of collateral in relation to the reinsurance arrangement i. We assume $Collateral_i = 0$.

According to SCR6.17 and SCR6.32, *Risk Mitigation* RM_i can be calculated as

$$RM_i = (SCR_{gross} - SCR_{net}) \cdot \frac{Recoverables_i}{Recoverables_{total}}$$

where

$SCR_{gross} =$ solvency capital requirements for underwriting risk calculated without reinsurance

$SCR_{net} =$ solvency capital requirements for underwriting risk taking reinsurance into account

$Recoverables_i =$ Best estimate recoverables from the reinsurance contract i.

$Recoverables_{total} =$ Best estimate recoverables from all reinsurance contracts together.

According to SCR6.55, Quantile factor q can be chosen as

$$q = \begin{cases} 3 & \text{if } \sqrt{V} \leq 5\% \cdot (LGD_1 + LGD_2 + \dots + LGD_N) \\ 5 & \text{else} \end{cases}$$

Calculation of the variance V is described in SCR6.7. We differentiate between counterparties belonging to different rating classes from AAA to CCC. We assume that in our case there are M rating classes for reinsurance counterparties. For each rating class j we first calculate y_j and z_j according to

$$y_j = LGD_1 + LGD_2 + \dots + LGD_{N_j}$$

$$z_j = LGD_1^2 + LGD_2^2 + \dots + LGD_{N_j}^2$$

where the sums are over all counterparties in the rating class j and N_j denotes the total number of counterparties in the rating class j .

Then, we calculate

$$\begin{aligned} V = & (u_{11} \cdot y_1 \cdot y_1 + u_{12} \cdot y_1 \cdot y_2 + \dots + u_{1M} \cdot y_1 \cdot y_M \\ & + u_{21} \cdot y_2 \cdot y_1 + u_{22} \cdot y_2 \cdot y_2 + \dots + u_{2M} \cdot y_2 \cdot y_M \\ & + \dots + u_{M1} \cdot y_M \cdot y_1 + u_{M2} \cdot y_M \cdot y_2 + \dots + u_{MM} \cdot y_M \cdot y_M) \\ & + (v_1 \cdot z_1 + v_2 \cdot z_2 + \dots + v_M \cdot z_M) \end{aligned}$$

Finally, the coefficients u_{ij} and v_i are calculated from the default probabilities valid for each different rating class

$$u_{ij} = \frac{p_i \cdot (1 - p_i) \cdot p_j \cdot (1 - p_j)}{(1 + \gamma) \cdot (p_i + p_j) - p_i \cdot p_j}$$

$$v_i = \frac{(1 + 2\gamma) \cdot p_i \cdot (1 - p_i)}{(2 + 2\gamma - p_i)}$$

where

$$\gamma = 0.25$$

and probabilities p_i are according to the following table

Rating	p_i
AAA	0,002%
AA	0,01%
A	0,05%
BBB	0,24%
BB	1,20%
B	6,04%
CCC or lower	30,41%

Annex A4

Indicative pricing of non-proportional reinsurance for the sample portfolio

The indicative pricing of non-proportional reinsurance in this paper is based on the following formalism.

For an arbitrary non-proportional reinsurance layer, we first introduce the parameter called *layer centroid* which has the following property: Expected loss covered by a layer A xs B with the liability limit A and retention B is equal to the liability level of the layer multiplied by the expected number of losses higher than the centroid of this layer.

$$Exp. Loss_{A \text{ xs } B} = A \cdot Exp. N_{loss > LC} \quad (A4.e1)$$

where $Exp. Loss_{A \text{ xs } B}$ denotes the expected loss covered by the layer A xs B and $Exp. N_{loss > LC}$ denotes the expected number of losses higher than the layer centroid.

From the above relation, we then derive for the net rate on line of the layer A xs B :

$$NetRoL_{A \text{ xs } B} \triangleq \frac{Exp. Loss_{A \text{ xs } B}}{A} = Exp. N_{loss > LC} \quad (A4.e2)$$

Therefore, the layer price parameter $NetRoL$ which is per definition equal to the expected loss covered by the layer divided by the layer liability limit can be calculated as the expected number of losses higher than the layer centroid.

For calculating $Exp. N_{loss > LC}$ we use the following relation

$$\begin{aligned} Exp. N_{loss > LC} &= Exp. N_{loss > LLL} \cdot P(loss > LC) \\ &= Exp. N_{loss > LLL} \cdot (1 - P(loss \leq LC)) \\ &= Exp. N_{loss > LLL} \cdot (1 - CDF(LC)) \end{aligned} \quad (A4.e3)$$

where $Exp. N_{loss > LLL}$ denotes the expected number of losses higher than lower loss limit (LLL) – the lower threshold of the loss severity distribution; $P(loss > LC)$ denotes the probability that the loss severity is higher than layer centroid LC according to the chosen loss severity distribution and $CDF(LC)$ denotes the cumulative distribution function of the chosen loss severity distribution. We further assume that $LC > LLL$ and that due to the fact that LLL is the lower threshold of loss severity distribution $P(loss \leq LLL) = CDF(LLL) = 0$.

For calculating $NetRoL$ according to (A4.e2) and (A4.e3), the value of the layer centroid is required. It can be shown that the following expression provides a good proxy:

$$Layer \text{ centroid of the layer } A \text{ xs } B \cong \sqrt{B \cdot (A + B)} \quad (A4.e4)$$

With the proxy (A4.e4), we can calculate $NetRoL$ of an arbitrary layer A xs B which expresses the expected loss covered by the layer divided by the liability limit A . The market price of layer can be expressed with help of the parameter called $GrossRoL$ which expresses the total of the expected loss covered by the layer and the costs allocated to the layer (costs of capital and management expenses) divided by the liability limit A .

$$GrossRoL_{A \times S \ B} = NetRoL_{A \times S \ B} + \frac{Costs}{A} \quad (A4.e5)$$

Instead of estimating the costs in (A4.e5), we apply an approach as adopted by capital markets when pricing catastrophe bonds. It estimates the *GrossRoL* as a product of *NetRoL* and the factor called *Multiple* which expresses the relation of the gross layer price (which includes the expected loss and the allocated costs) to the pure expected loss.

$$GrossRoL_{A \times S \ B} = Multiple \cdot NetRoL_{A \times S \ B} \quad (A4.e6)$$

Expected loss covered by the layer *A xs B* and the reinsurance premium can be calculated from the net and gross rate on line according to the following:

$$Exp. Loss_{A \times S \ B} = A \cdot NetRoL_{A \times S \ B} \quad (A4.e7)$$

$$RI \ Premium_{A \times S \ B} = A \cdot GrossRoL_{A \times S \ B} \quad (A4.e8)$$

Often, the expected loss and the price of an XL layer are expressed as rate on GNPI (gross net premium income) – premium retained under preceding proportional reinsurance (if any):

$$NetRate_{A \times S \ B} = Exp. Loss_{A \times S \ B} / GNPI \quad (A4.e9)$$

$$GrossRate_{A \times S \ B} = RI \ Premium_{A \times S \ B} / GNPI \quad (A4.e10)$$

For the loss severity we chose censored Pareto distribution with the following cumulative distribution function

$$CDF_{Pareto}(x) = \frac{LLL^{-\alpha} - x^{-\alpha}}{LLL^{-\alpha} - ULL^{-\alpha}} \quad (A4.e11)$$

where α denotes the shape parameter of the Pareto distribution, *LLL* and *ULL* denote the lower and the upper loss limit respectively. The following Table A4.1 summarizes the chosen parameter of Pareto distribution per line of business. The values of Pareto Alpha in column C were chosen according to market benchmarks.

Table A4.1: Parameters of the loss severity distribution (Pareto).

Line of business	A. Lower Loss Limit (LLL)	B. Upper Loss Limit (ULL)	C. Pareto Alpha
Motor vehicle liability insurance	250	100,000	2.20
Other motor insurance	250	100,000	1.85
Marine, aviation and transport insurance	250	100,000	1.50
Fire and other damage to property insurance			
Fire non natural catastrophe per risk	250	100,000	1.40
Fire natural catastrophe per event (EQ)	250	100,000	0.80
General liability insurance	250	100,000	1.85

The following Table A4.2 provides the estimates for $Exp. N_{loss > LLL}$ – the expected number of losses higher than *LLL*, cf. (A4.e3). The values in column D are the assumed frequencies p.a. per € 1 m premium. For simplicity reasons, we assumed the same frequency for each

line of business, in-line with market benchmarks. The values in column F (except for fire earthquake) were obtained by multiplying the premiums in column E by the frequencies per € 1m premium (Table A4.1, column D). Obviously, the frequency of the earthquake loss does not depend on premium. It was estimated directly assuming the return period of 10 years for the loss higher than the lower loss threshold which translates into the frequency of 0.1.

Table A4.2: Loss frequencies

Line of business	D. Exp. number of losses xs LLL per 1m premium p.a.	E. Gross premium (m €)	F = D * E Exp. number of losses xs LLL p.a.
Motor vehicle liability insurance	0.2	20.0	4.0
Other motor insurance	0.2	5.0	1.0
Marine, aviation and transport insurance	0.2	5.0	1.0
Fire and other damage to property insurance			
Fire non natural catastrophe per risk	0.2	15.0	3.0
Fire natural catastrophe per event (EQ)			0.1
General liability insurance	0.2	5.0	1.0

Based on the formalism presented above (A4.e1 – A4.e11) and the parameters of loss severity and loss frequency distributions summarized in Tables A4.1 and A4.2, we estimated the prices of all XL layers throughout the paper. Below we provide calculation details for the layers of the original reinsurance structure, cf. Section 2.1. All other layers were priced analogously.

Table A4.3 summarizes the parameters of the original reinsurance portfolio.

Table A4.3: Parameters of the original reinsurance portfolio.

Line of business	G. Proportional cession	H = E * (1 - G) Premium retained under prop. RI (m €)	I. Limit of XL reinsurance	J. Retention of XL reinsurance
Motor vehicle liability insurance	0%	20.0	9,500	500
Other motor insurance	0%	5.0	9,500	500
Marine, aviation and transport insurance	50%	2.5	24,500	500
Fire and other damage to property insurance				
Fire non natural catastrophe per risk	30%	10.5	20,500	500
Fire natural catastrophe per event (EQ)	30%	10.5	69,500	500
General liability insurance	0%	5.0	4,500	500

Based on the layer parameters provided in Table A4.3, columns I and J, we then calculated layer centroids and the expected number of losses higher than layer centroid, cf. Table A4.4.

Table A4.4: Layer centroids and expected number of losses higher than layer centroid

Line of business	$K = A * (1 - G)$ Lower loss limit after prop RI (LLL)	$L = B * (1 - G)$ Upper loss limit after prop RI (ULL)	$M = \text{sqrt}(J * (I + J))$ Layer centroid	$N = F * (1 - \text{CDF}(M))$ Exp. number of losses xs layer centroid
Motor vehicle liability insurance	250	100,000	2,236	0.032
Other motor insurance	250	100,000	2,236	0.017
Marine, aviation and transport insurance	125	50,000	3,536	0.007
Fire and other damage to property insurance				
Fire non natural catastrophe per risk	175	70,000	3,240	0.050
Fire natural catastrophe per event (EQ)	175	70,000	5,916	0.005
General liability insurance	250	100,000	1,581	0.033

For calculating column M, we used (A4.e4) and for calculating column N (A4.e3) and (A4.e11). Please notice that in (A4.e11) the lower and upper loss limits from Table A4.4 columns K and L were used. These loss limits were obtained from the original loss limits (Table A4.1, columns A and B) through applying the retention ratio under the proportional reinsurance.

Table A4.5 provides the results for *NetRoL* and *GrossRoL* per line of business.

Table A4.5: Net and gross RoL

Line of business	O = N Net RoL	P. Multiple	Q = O * P Gross RoL
Motor vehicle liability insurance	3.2%	1.5	4.8%
Other motor insurance	1.7%	1.5	2.6%
Marine, aviation and transport insurance	0.7%	1.5	1.0%
Fire and other damage to property insurance			0.0%
Fire non natural catastrophe per risk	5.0%	1.5	7.5%
Fire natural catastrophe per event (EQ)	0.5%	2.0	1.0%
General liability insurance	3.3%	1.5	4.9%

For calculating column O, we used (A4.e2). The Multiples in column P were assumed based on market benchmarks. For calculating column Q, we used (A4.e6).

Finally, Table A4.6 calculates the expected loss covered by the XL layers and the reinsurance premium as € figures as well as the net and gross rates on GNPI.

Table A4.6: Expected covered loss, reinsurance premium, net and gross rate on GNPI

Line of business	$R = O * I$ Exp. Loss covered by the XL layer	$S = Q * I$ XL RI premium	$T = R / H$ Net rate on GNPI	$U = S / H$ Gross rate on GNPI
Motor vehicle liability insurance	306	460	1.5%	2.3%
Other motor insurance	165	247	3.3%	4.9%
Marine, aviation and transport insurance	160	240	6.4%	9.6%
Fire and other damage to property insurance				
Fire non natural catastrophe per risk	1,020	1,530	9.7%	14.6%
Fire natural catastrophe per event (EQ)	361	722	3.4%	6.9%
General liability insurance	148	222	3.0%	4.4%

For calculating column R, we used (A4.e7), for column S (A4.e7), for column T (A4.e9) and for column U (A4.e10). In the paper, these results were used in Tables 2.3.1 and 2.3.9, cf. Sections 2.3.1 and 2.3.4.