Aging, Social Security Design, and Capital Accumulation

Antoine Dedry

Harun Onder

Pierre Pestieau

WORLD BANK GROUP

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Abstract

This paper analyzes the impact of aging on capital accumulation and welfare in a country with a sizable unfunded social security system. Using a two-period overlapping generation model with endogenous retirement decisions, the paper shows that the type of aging and the type of unfunded social security system are important in understanding this impact.

The analysis compares two types of demographic changes, declining fertility and increasing longevity; three types of pensions, defined contributions, defined benefits, and defined annuities; as well as mandatory and optimal retirement systems to investigate the differences in implications of aging.

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Aging, Social Security Design, and Capital Accumulation^{*}

Antoine Dedry, Harun Onder, and Pierre Pestieau[†]

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[†]Dedry: University of Liège, Department of Economics, e-mail: Antoine.dedry@icloud.com; Onder: Macroeconomics and Fiscal Management Global Practice, The World Bank, e-mail: honder@worldbank.org, and Pestieau: Université Catholique de Louvain, CORE, e-mail: p.pestieau@ulg.ac.be.

1 Introduction

Demographic aging poses a major challenge to all industrialized economies and a large number of developing countries. According to UN projections, total world population will increase by 40% and median age on the planet will increase by 7.8 years within the next four decades. Compared to the recent history, these changes represent a significant slow-down in population growth and a considerable acceleration in aging.

Although an increase in the average age is a common trend around the world, the factors that lead to such changes vary across countries. In the short and medium term these may be led by transitory events such as out-migration of young population, wars or aging of past baby-boom generations. However, in the long-term, it can be traced back to decreases in fertility rates and increases in longevity, albeit at different magnitudes of importance in different economies.

Economic implications of demographic aging are complex, and they are not always well understood in public discussions. Some consequences are clearly unfavorable: aging, whether it is driven by a decrease in fertility or an increase in longevity, pushes the old-age dependency rate up, i.e. a larger group of elderly people relative to economically active population. This will, in turn, increase the pressure on financial balances of unfunded pensions. In contrast, some consequences of aging are perceived to be positive: if the decline in fertility outweighs the increase in longevity, total population would decrease. This outcome may be welcomed by some on the basis of environmental concerns. Finally, there are some ambiguous consequences. An example is the effect of aging on capital accumulation, a key determinant of growth, which we investigate in this paper.

Studying the effect of aging on capital accumulation is particularly difficult when a large number of discretionary policy choices can affect the outcomes. In this paper, we use a two period overlapping generation model to show that the effect of aging on capital accumulation and welfare depend on: i) the type of aging, i.e. decreasing fertility or increasing longevity, ii) the type of unfunded social security system, i.e. defined contribution (DC), defined benefit (DB), or defined annuities (DA), and finally, iii) the regulation of the retirement age, i.e. mandatory retirement vs. *laissez-faire*.

To fix these ideas, we set up an economic environment where each individual lives two periods. The first period of her life has a unitary length, while the second one has a variable longevity. In the first period, the individual works and earns a wage equal to her marginal productivity net of social security contributions if any. This income is then devoted to consumption in the first period and saving for future consumption. In the second period, she works for a fraction of her remaining lifetime, where the work

duration is determined by the balance between the marginal income and disutility created by the work. In the case of mandatory retirement, however, the optimal retirement choice may be overwritten and she may be forced to retire earlier than she would like to. In the end, the second period consumption is equal to the earnings from within period work, savings from first period with interest earnings, and finally pension benefits from the unfunded social security system.

This structure enables us to elaborate on our main results by introducing a number of institutional and demographic factors *vis a vis* the standard Diamond case, where individuals do not work in the second period of their life time and there is no PAYG pension system. First, we allow work in the second period of life and investigate how a mandatory early retirement rule affects the outcome. Second, we consider different types of unfunded social security systems in order to see how incentives respond to changes in demography under different pension system obligations and entitlements. Third, and finally, we also investigate these effects under different aging profiles, i.e. fertility driven vs. longevity driven changes in age composition.

The main contribution of this study is, then, to show the incidence of these three factors on the effects of aging on capital accumulation and welfare. In the standard Diamond case, an increase in fertility decreases capital accumulation in the absence of a PAYG pension system, as capital is diluted with more workers. In our framework, this depressive effect is reinforced if the country has a DC pension system. In contrast, it is weakened or possibly reversed with DB or DA pensions. Similar results are also derived with increasing longevity. A small increase in longevity has a fostering effect on capital accumulation in the standard case. Introducing PAYG pensions and possibility of work in the second period, however, diminishes and potentially reverses the fostering effect.

The economics literature comprises a large number of studies devoted to understanding the effects of demographic aging in different settings. These could be classified on the basis of numerous criteria: explicit recognition of the distinction between different sources of aging, e.g. longevity and fertility changes, consideration of different social security systems, and characterization of growth in exogenous or endogenous settings. We will not provide an exhaustive review of this large field. However, a subset of these studies which investigate how institutional factors and behavioral responses may affect the impact of aging on capital accumulation is more relevant for our purposes.

An interesting discussion on the effect of longevity increase on growth is provided by Bloom et.al. (2007). The authors point out that, in theory, improvements in healthy life expectancy should generate increases in the average age of retirement, with little effect on savings rates. In many countries, however, retirement incentives in social security pro-

grams prevent retirement ages from keeping pace with changes in life expectancy, leading to an increased need for life-cycle savings. Using a cross-country panel of macroeconomic data, then, the paper finds that increased longevity raises aggregate savings rates in countries with universal pension coverage and retirement incentives. Similarly, Bloom et. al. (2003) show that aging leads to more capital accumulation even if retirement is endogenous. Echevarria (2004) reaches the same conclusion. Kalemli-Ozcan *et.al.* (2000) show that the positive effect of mortality decline on capital accumulation is made larger if education decisions are endogenous.

De la Croix and Licandro (1999) and Zhang et al. (2001, 2003) argue that the effect of increasing longevity depends on its initial level. For low levels of life expectancy the effect is positive but it can turn negative for high levels. Similarly, Miyazawa (2006) also shows that the effect of an increase in longevity on economic growth has a hump-shaped pattern. This is the result of a two-effect system. First, higher longevity increases the aggregate saving rate directly by increasing precautionary saving for the prolonged retirement and indirectly by increasing the accidental bequests (bequest-wage ratio is important because the higher income group has a higher propensity to save). Second, it reduces the frequency of accidental bequests, which implies that the population share of the higher income group decreases. This leads to a reduction in aggregate savings. The relative shares of these factors change over the aging horizon. This is also true for the income inequality (first positive, then negative). Kinugasa and Mason (2006) provide empirical support to shows that an increase of wealth across countries is likely with mortality decline.

Among the studies that link the impact of aging with social security systems, Ito and Tabata (2008) find that the unfunded social security system provides a sufficient mechanism to have such a hump shaped relationship between longevity and per capita output. Tabata (2014) looks at the effect of a shift from a DB to a DC PAYG pension on growth. He shows that this shift is growth enhancing and alleviates the cost of aging. Heijdra, B. and J. Mierau (2011) also compare the relative effects of DB and DC PAYG pensions on economic growth with aging. They show that the DC formula fares better than the DB one in facilitating growth. They also show that raising the retirement age as a response to an increase in longevity dampens the growth gains. The analysis in this paper provides a comparison of several different institutional settings, i.e. different social security systems and retirement age policies, and types of aging in a unified framework. Therefore, it provides a consistent survey of the welfare effects of demographic aging under various conditions.

The rest of the paper is organized as follows. In section 2, we present the basic model and the main results for an economy that consists of identical individuals with a defined

contribution pension system. Section 3 is devoted to comparative statics where we investigate the change in capital accumulation and welfare for all pension systems, retirement schemes, and aging types. Fourth and fifth sections present static and dynamic simulations, respectively. Finally, the last section offers some concluding remarks.

2 Basic Model

We use a standard two-period overlapping generation model. An individual who belongs to generation t lives in two periods: t and $t + 1$. The first period of her life has a unitary length, while the second one has a length $\ell \leq 1$, where ℓ reflects variable longevity.

In the first period, the individual works and earns a wage, w_t , which is devoted to the first-period consumption, c_t , saving, s_t , and pension contribution, τ . In the second period, she works an amount of time $z_{t+1} \leq \ell \leq 1$ and earns $z_{t+1}w_{t+1}$. These earnings, together with the proceeds of savings $R_{t+1}s_t$ and the PAYG pension p , finance the second period consumption d_{t+1} .

We assume that working in the second period z_{t+1} implies a disutility defined in monetary terms $v(z_{t+1}, \ell)$, where $\frac{\partial v}{\partial z} > 0$, $\frac{\partial^2 v}{\partial v^2} > 0$ are imposed for the existence of a unique solution. In addition, disutility from working in the second period of life is a decreasing function of longevity, i.e. $\frac{\partial v}{\partial \ell} < 0$, which reflects the idea that an increase in longevity fosters later retirement. Note that, for simplicity, earnings in the second period of life is not taxed. Intuitively, the end of the first period can be interpreted as the statutory age of retirement, unless otherwise indicated by an explicit mandatory retirement age. Any savings in funded social security system is not modeled explicitly, and it is assumed to be identical to other savings. Thus, the pension contribution parameter τ measures the relative size of the unfunded pensions. In other words, $\tau = 0$ implies that the whole pension system is funded.

Denoting by $u(\cdot)$ the utility function for consumption c or d , and U the lifetime utility, the problem of an individual of generation t is:

$$\max U_t = u(w_t - \tau - s_t) + \beta \ell u\left(\frac{w_{t+1}z_{t+1} + R_{t+1}s_t + p - v(z_{t+1}, \ell)}{\ell}\right) \quad (1)$$

where $p = \tau(1 + n)$ is the pension benefit in period $t + 1$ and β is the time discount factor. The gross rate of population growth $(1 + n)$ is equivalent to the number of children per individual in this set up. The argument of second period utility is net amount of resources

then available divided by the length of the second period.¹

The first order conditions for life time utility maximization are simply given by:

$$v'_{z_{t+1}}(z_{t+1}, \ell) = w_{t+1} \quad (2)$$

$$\beta R_{t+1} u'(d_{t+1}) - u'(c_t) = 0 \quad (3)$$

where c_t and d_{t+1} denote the first and second period consumption. The first condition (2) shows that the marginal disutility from second period work needs to be equal to the wage rate at the optimum. The second condition is the consumption Euler equation, and it shows that the individual cannot gain further utility by reallocating consumption between periods. In order to be able to show some of our results analytically, we will use simple functional forms for $u(\cdot)$ and $v(\cdot)$. Accordingly, we assume that the period utility function is logarithmic $u(x) = \ln x$, and the monetary disutility function is quadratic in its main argument $v(x) = x^2/2\gamma\ell$. One clearly sees from the latter functional form that the disutility of working longer can be mitigated by an increase in longevity. We can now rewrite the problem of the individual as the following:

$$U_t = \ln(w_t - \tau - s_t) + \beta \ln \left(\frac{w_{t+1}z_{t+1} + R_{t+1}s_t - z_{t+1}^2/2\gamma\ell + p}{\ell} \right) \quad (4)$$

The first order condition with respect to z_{t+1} yields :

$$z_{t+1} = z_{t+1}^* = \gamma\ell w_{t+1} \quad (5)$$

where, an asterisk (*) denotes an optimal solution.² Using this optimality condition, and incorporating $p = \tau(1 + n)$, we can then get an explicit solution for the optimal saving rates:

$$s_t = \frac{\beta\ell}{1 + \beta\ell} w_t - \frac{\gamma\ell w_{t+1}^2}{2R_{t+1}(1 + \beta\ell)} - \tau \left(\frac{\beta\ell}{1 + \beta\ell} + \frac{1 + n}{(1 + \beta\ell)R_{t+1}} \right) \quad (6)$$

¹Suppose T_1 (T_2) is the number of years of the first (second) period, where $T_1 > T_2$. The life time utility would then be: $U = T_1 u[(w - \tau - s)/T_1] + \beta T_2 u((wz + Rs + p - v(z, T_2))/T_2)$, where we normalize T_1 and T_2 so that $T_1 = 1$ and $T_2 = l$.

²Note that we assume that lifetime utility is increasing in longevity: $\frac{\partial U}{\partial l} = \beta \left[u(d) - u'(d)d - u'(d)\frac{\partial z(z, l)}{\partial l} \right] > 0$, which is satisfied if $\frac{u'(d)d}{u(d)} < 1$. Intuitively speaking one more year of life is worth living. With the functional forms we use, this condition is reduced to $\frac{\partial U}{\partial l} = \beta \left[\log(d) - 1 + \frac{\gamma w^2}{d} \right] > 0$.

In many countries, z is not the outcome of a choice without a distortion. Through an array of programs, workers are induced to retire at ages different from what they would choose in the absence of these programs. We consider a case where the workers are induced to retire earlier than they wish to do so, and denote this induced early retirement by $\bar{z}$.³ In the case of this mandatory early retirement, we rewrite equations (5) and (6) as follows :

$$z_{t+1} = \bar{z}$$

$$s_t = \frac{\beta\ell}{1+\beta\ell}w_t - \frac{\bar{z}}{R_{t+1}(1+\beta\ell)}(w_{t+1} - \bar{z}/2\gamma\ell) - \tau \left(\frac{\beta\ell}{1+\beta\ell} + \frac{1+n}{(1+\beta\ell)R_{t+1}} \right) \quad (7)$$

We now turn to the production side of the economy. The technology is characterized by a Cobb-Douglas production function:

$$Y_t = F(K_t, N_t) = AK_t^\alpha N_t^{1-\alpha} \quad (8)$$

where K is the stock of capital, A is a productivity parameter, and N is the labor force. We distinguish the labor force N_t from the size of generation t , L_t . The labor force comprises the young population of generation t and the labor force participation from the old generation $t-1$. Incorporating the population growth, $L_t = L_{t-1}(1+n)$, the labor force can then be written as $N_t = L_t + L_{t-1}z_t = L_{t-1}(1+n+z_t)$. In comparison, total population at time t is :

$$L_t + \ell L_{t-1} = L_{t-1}(1+\ell+n).$$

Denoting $K_t/N_t \equiv k_t$ and $Y_t/N_t \equiv y_t$, we obtain the income per worker (and not per capita):

$$y_t = f(k_t) = Ak_t^\alpha$$

³An alternative specification could be that second period labor is subject to a proportional tax θ whose proceeds are returned to the old workers. Their problem would be to choose z such as to maximize : $wz(1-\theta) + T - v(z, \ell)$. With $T = \theta wz$ and $v = z^2/2\gamma\ell$, this yields $z = \gamma\ell w(1-\theta)$. In the case of optimal retirement with no distortions, $z = z^* = \gamma\ell w$. In the case of induced early retirement, $z = \bar{z} = \gamma\ell w(1-\theta)$, where θ is chosen such as to generate $\bar{z} < z^*$.

Factors of production are paid according to their marginal contributions :

$$R_t = f'(k_t) = A\alpha k_t^{\alpha-1} \quad (9)$$

$$w_t = f(k_t) - f'(k_t)k_t = (1 - \alpha) A k_t^\alpha \quad (10)$$

the equilibrium conditions in the labor and capital markets are as follows:

$$N_t = L_{t-1} (1 + n + z_t) \quad (11)$$

$$K_{t+1} = L_t s_t \quad (12)$$

where the latter expression reflects the fact that capital is assumed to depreciate completely after each period. Although this assumption arises from convenience, it is not unrealistic considering the fact that a period denotes several decades in calendar. Using the optimality condition for savings derived before, the latter expression can be rewritten as follows :

$$\begin{aligned} G_t \equiv & (1 + n + z_{t+1}) k_{t+1} - \frac{\beta\ell}{1 + \beta\ell} A(1 - \alpha) k_t^\alpha + \tau \left(\frac{\beta\ell}{1 + \beta\ell} + \frac{(1 + n) k_{t+1}^{1-\alpha}}{A\alpha (1 + \beta\ell)} \right) \\ & + \frac{z_{t+1} k_{t+1}^{1-\alpha}}{(1 + \beta\ell) A\alpha} \left(A(1 - \alpha) k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma\ell} \right) = 0 \end{aligned} \quad (13)$$

which explicitly defines the dynamic behavior of capital stock. Note that the standard (Diamond) case with no social security and work in second period of life can be deduced by shutting down the corresponding sections, $z = \tau = 0$, which generates the following:

$$G_t \equiv (1 + n) k_{t+1} - \frac{\beta\ell}{1 + \beta\ell} A(1 - \alpha) k_t^\alpha \quad (14)$$

Comparing (13) and (14), we observe two main differences. First, the third term on the right hand side of (13) denotes the double burden that the PAYG imposes to saving. Second, the fourth term reflects the double effect of working in the second period: a distortionary effect if z is not optimal and a saving inducement if $z < z^*$.

In equations (6) and (7) we assumed a pension system that relies on a defined contribution (DC) formula in which the tax $\bar{\tau}$ is given and thus the benefits p has to follow through based on demographic shifts. Two alternative systems can also be considered. The first one provides a defined benefit (DB) $\bar{p}$ over the second period. In this case, the individual receives a predetermined lump sum payment after retirement, and contribution rate is endogenously determined. The other one is a scheme which offers constant

Table 1: Different Social Security and Retirement Regimes

Type of Social Security System		Retirement Age Regulation Scheme
Case-1	Defined Contribution	Mandatory Early Retirement
Case-2	Defined Benefit	Mandatory Early Retirement
Case-3	Defined Annuity	Mandatory Early Retirement
Case-4	Defined Contribution	Optimal Retirement
Case-5	Defined Benefit	Optimal Retirement
Case-6	Defined Annuity	Optimal Retirement

Notes: The optimal retirement is given by $z^* = \gamma \ell w$, and the mandatory early retirement is given by $z = \bar{z} < z^*$.

annuities $\bar{a}$ (DA) during retirement.⁴ The three revenue constraints that these systems imply are as follows:

$$\begin{aligned}
 DC : \bar{\tau}(1+n) &= p \\
 DB : \tau(1+n) &= \bar{p} \\
 DA : \bar{a}(\ell - z) &= \tau(1+n),
 \end{aligned}$$

where an upper bar denotes the defined variable. For example, $\bar{a}$ is the defined annuity and τ has to adjust to variations in z , ℓ and n in this case. Note that for each type of pension system, the individual utility has to adjust accordingly.

We have so far identified two major dimensions which characterize a social security system : is it DB, DC, or DA and is there mandatory early retirement or not? Altogether, these two dimensions provide six different ways to describe the equation (13). The six cases are presented on Table 1, and the corresponding equations for G_{it} are provided in the appendix.

Using these six alternative specifications, we now turn to the comparative statics of the problem. But before let us state that we assume that the dynamics of capital accumulation (eq(13)) lead to a unique and stable equilibrium, which implies that $0 < \frac{\partial k_{t+1}}{\partial k_t} < 1$. This condition implies that in the steady state $G_k = \frac{\partial G}{\partial k} > 0$. With the functional forms used in this paper, these conditions are fulfilled in a Diamond setting, but not necessarily in ours.

⁴With defined benefits, there are two ways of exiting from the pension system at the time of retirement: either through annuities or by cashing some capital. Most public DB systems comprise an exit in annuities. Some DB private systems, on the other hand, provide the option to choose between the two types of exits.

3 Comparative Statics

In this section, we investigate the comparative statics for the six alternative cases of social security systems identified in the previous section. Our main aim is to elaborate on the behavior of capital accumulation when the economy experiences aging due to lower fertility or higher longevity.

3.1 Mandatory Early Retirement

We begin by showing the impact of a decrease in fertility in a mandatory early retirement system:

$$DC : \quad \Psi \frac{\partial k^1}{\partial n} = -k - \frac{\bar{\tau} k^{1-\alpha}}{A\alpha(1+\beta\ell)} < 0 \quad (15)$$

$$DB : \quad \Psi \frac{\partial k^2}{\partial n} = -k + \frac{\bar{p}\beta\ell}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (16)$$

$$DA : \quad \Psi \frac{\partial k^3}{\partial n} = -k + \frac{\bar{a}(\ell-z)\beta\ell}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (17)$$

where $\Psi = \left(\frac{\partial G}{\partial k}\right)^{-1} > 0$, and superscripts denote the type of social security as defined in Table 1. In a standard case (Diamond), an increase in fertility has a depressive effect on capital accumulation in the absence of a PAYG pension system. This is shown by the first term on the right hand side of each equation above. This depressive effect is reinforced with a DC pension system as shown by the negative second term in (15), but it is weakened or possibly reversed with DB or DA pensions as shown by positive second terms in (16) and (17). The explanation is quite intuitive. With a DC system, an increase in fertility implies an increase in pension, which discourages saving. With a DB or DA system the pension level is kept constant and thus the contribution rate decreases, which fosters saving. With a too generous pension, an increase in fertility could even lead to an increase in capital.

Next, we turn to the impact of an increase in longevity on equilibrium capital per

worker in an induced early retirement system:

$$DC : \quad \Psi \frac{\partial k^1}{\partial \ell} = \frac{1}{(1 + \beta \ell)^2} \left[Ak^\alpha (1 - \alpha) \beta - \beta \bar{\tau} \left(1 - \frac{k^{1-\alpha} (1 + n)}{A\alpha} \right) - \Omega \right] \geq 0 \quad (18)$$

$$DB : \quad \Psi \frac{\partial k^2}{\partial \ell} = \frac{1}{(1 + \beta \ell)^2} \left[Ak^\alpha (1 - \alpha) \beta - \beta \bar{p} \left(\frac{1}{1 + n} - \frac{k^{1-\alpha}}{A\alpha} \right) - \Omega \right] \geq 0 \quad (19)$$

$$DA : \quad \Psi \frac{\partial k^3}{\partial \ell} = \frac{1}{(1 + \beta \ell)^2} [Ak^\alpha (1 - \alpha) \beta - \bar{a} \Theta - \Omega] \geq 0 \quad (20)$$

where the last term in brackets is defined by $\Omega = \frac{\bar{z} k^{1-\alpha}}{2A\alpha\gamma\ell^2} [\bar{z}(1 + 2\beta\ell) - 2\gamma\beta Ak^\alpha \ell^2 (1 - \alpha)]$ and $\Theta = \beta(\ell - \bar{z})(\frac{1}{1+n} - \frac{k^{1-\alpha}}{A\alpha}) + (1 + \beta\ell)(\frac{k^{1-\alpha}}{A\alpha} + \frac{\beta\ell}{1+n})$. The term first expression (Ω) reflects the negative impact that $\bar{z}$ has on capital accumulation; it comprises the direct effect of increasing mandatory retirement and the allocative effect of reducing the gap between z^* and $\bar{z}$. The second expression (Θ), on the other hand, reflects the negative effect of defined annuities on capital accumulation. Overall, the above inequalities show that a small increase in longevity has a fostering effect on capital accumulation in the absence of pension and work in the second period. When these features are introduced, however, this effect is diminished and could even be reversed, especially in the case of defined annuities. An increase in longevity magnifies the cost of mandatory retirement and the cost of the pension system in all cases.

Note that here we make the assumption of dynamic efficiency. This implies that the marginal productivity of capital is higher than the rate of fertility, $\alpha Ak^{\alpha-1} = R > 1 + n$, and it has the known effect that social security depresses capital accumulation and welfare (See Aaron (1966)). One also observes that the earlier the mandatory age of retirement and the higher the disutility of working late, the larger the level of saving.

3.2 Optimal Retirement

We now relax the early retirement assumption, and analyze the impact of aging on capital accumulation when retirement is chosen optimally. The analysis here shows that, compared to the case with early retirement, the ability to adjust the retirement age optimally

leads to less distortion in equilibrium, but it also diminishes the incentives for saving.

$$DC : \quad \Psi \frac{\partial k^4}{\partial n} = -k - \frac{\bar{\tau} k^{1-\alpha}}{A\alpha(1+\beta\ell)} < 0 \quad (21)$$

$$DB : \quad \Psi \frac{\partial k^5}{\partial n} = -k + \frac{\bar{p}\beta\ell}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (22)$$

$$DA : \quad \Psi \frac{\partial k^6}{\partial n} = -k + \frac{\bar{a}\beta\ell^2(1-\gamma D)}{(1+\beta\ell)(1+n)^2} \geq 0 \quad (23)$$

Similar to the case with early retirement, the PAYG pension system reinforces the depressive effect of an increase in fertility on capital accumulation in the DC case and weakens or possibly reverses it in the DB or DA cases. Turning to the effect of longevity when z is endogenous, we have:

$$DC : \quad \Psi \frac{\partial k^4}{\partial \ell} = \frac{1}{(1+\beta\ell)^2} \left[Ak^\alpha(1-\alpha)\beta - \beta\bar{\tau} \left(1 - \frac{k^{1-\alpha}(1+n)}{A\alpha} \right) - \Phi \right] \geq 0 \quad (24)$$

$$DB : \quad \Psi \frac{\partial k^5}{\partial \ell} = \frac{1}{(1+\beta\ell)^2} \left[Ak^\alpha(1-\alpha)\beta - \beta\bar{p} \left(\frac{1}{1+n} - \frac{k^{1-\alpha}}{A\alpha} \right) - \Phi \right] \geq 0 \quad (25)$$

$$DA : \quad \Psi \frac{\partial k^6}{\partial \ell} = \frac{1}{(1+\beta\ell)^2} \left[Ak^\alpha(1-\alpha)\beta - Dk\gamma \left((1+\beta\ell)^2 + \frac{\bar{a}}{2A\alpha k^\alpha} + \frac{(1-\alpha)}{2\alpha} \right) - \chi \right] \geq 0 \quad (26)$$

where the last expressions are defined by $\Phi = \frac{Ak^{1+\alpha}(1-\alpha)\gamma}{2\alpha} [2\alpha(1+\beta\ell)^2 + (1-\alpha)]$, and $\chi = \bar{a}(1-D\gamma) \left[\left(\frac{k^{1-\alpha}}{A\alpha} + \frac{\beta\ell}{1+n} \right) + \beta\ell \frac{(1+\beta\ell)}{1+n} \right]$, whereas $D = (A(1-\alpha)k^\alpha - \bar{a}) > 0$. The first term (Φ) reflects the effect of endogenous retirement on capital accumulation as measured by gamma, the parameter of preference for working late. The second term (χ) and D reflect also the incidence of retirement in the case of DA with two components: a larger z means less saving and at the same time it may imply an additional tax burden if $l - z^*$ increases with l . These derivatives show that both pension and retirement terms weaken the positive effect of an increase in ℓ on equilibrium value of k for the three types of pensions systems. With endogenous retirement age, retirement age increases with longevity particularly for high level of longevity. This has for consequence that saving does not increase with longevity as much as with mandatory early retirement. We summarize these findings in Table 2.

Table 2: Summary Results of Comparative Statics: The Effect of Aging on Equilibrium Capital Per Worker

	Standard Case	Defined Contribution	Defined Benefit	Defined Annuity
		<i>Mandatory Early Retirement</i>		
Decrease in Fertility	> 0	> 0	≤ 0	≤ 0
Increase in Longevity	> 0	≤ 0	≤ 0	≤ 0
		<i>Optimal Retirement</i>		
Decrease in Fertility	> 0	> 0	≤ 0	≤ 0
Increase in Longevity	> 0	≤ 0	≤ 0	≤ 0

As we consistently assume dynamic efficiency, all things being, equal utility under optimal retirement is not necessarily higher than under mandatory retirement. This is mainly because mandatory early retirement induce higher saving and capital accumulation as desired consumption in the second period of life time cannot be financed by extending the work hours. As a result, mandatory early retirement presents a case that is closer to the golden rule than the optimal retirement. This is a standard second-best problem where a distortion makes a second distortion desirable.

4 Simulations

The analysis so far has shown that demographic change has different implications for capital accumulation under alternative PAYG systems. Analytically, these results are sufficient to show that the impacts are different quantitatively. In order to better grasp these effects, however, we employ a numerical example. To this effect, we calibrate the equilibrium using common parameter values from the literature to simulate the equilibrium profiles for the agent's lifetime utility U and the capital per worker k with different values of fertility n and longevity ℓ . Figure 1 shows these values and the results in the spirit of static equilibrium.

We start with the interpretation of the case of declining fertility. Note that at the starting point, namely at the highest level of fertility, we impose a condition that the mandatory age of retirement is not distortionary, e.g. it is set at the optimal retirement age. This explains the same level of capital and generational utility at that point. In all three pension regimes considered, DC, DB and DA, the capital stock increases as fertility decreases. This was expected for DC but is not guaranteed for DB and DA as comparative statics in the previous section suggested. In fact, with social security contributions, which reflects the fact that the distortionary nature of the social security system is greater, a decrease in capital per worker in DB and DA systems is possible. Nevertheless, even with the

Figure 1: Equilibrium Values of Lifetime Utility and Capital Per Worker

Notes:

The parameter values used in the simulations above are as follows: In all cases $A = 10$, $\alpha = 0.33$, $\beta = 0.25$, and $\gamma = 0.15$. These reflect commonly used calibrations such as the share of capital in the economy and time preferences. In addition, the following assumptions were made when needed: $a = 1.6$, $\bar{z} = 0.394$ in defined annuity case with mandatory retirement, and $\bar{z} = 0.579$ in defined benefit and defined contribution cases. These values are chosen to make the initial conditions and changes comparable with alternative social security systems. Longevity and fertility rates are kept constant in fertility and longevity shock scenarios, with values $l = 0.9$ and $n = 0.2$, respectively.

parameter values we use, not surprisingly, the increase in capital is higher in DC than the one with other two systems. With a monotonic increase in capital, we expect that wage and thus the optimal age of retirement to increase. This explains why in these three graphs the capital stock with mandatory retirement (dashed black line) dominates the capital stock under optimal retirement (solid black line). In other words, as n decreases, the labor distortion implied by mandatory retirement increases.

The differences between utility profiles under different social security systems are stronger as compared to differences in capital. A decrease in fertility leads to an increase in lifetime utility with DC, a decrease with DB and first an increase and then a decrease (hump shaped) with DA. In the case of DA, there is not much difference between capital accumulations under optimal and mandatory retirement systems. However, utility is significantly higher in the optimal retirement case. This shows that the distortion imposed by mandatory early retirement dominates the gains in capital accumulation (dynamic optimality). In contrast, the additional capital accumulation induced by restraining second period work under the mandatory early retirement system dominates the inefficiency of the labor market and thus the utility is higher with mandatory retirement in DB and DC systems. Finally, note that utility decreases with lower fertility rates in the case of DA with mandatory retirement and in the case of DB with the both types of retirement age regulation. In these cases, a decrease in fertility leads to an increase in the tax burden, which, in turn, affects the first period utility of the individuals by reducing their consumption.

We next turn to aging due to an increase in longevity. First of all, there is no difference between DC and DB in this case as longevity does not affect social security taxes and benefits. We thus only interpret two figures. Note that capital per worker is increasing with longevity in both mandatory retirement and optimal retirement cases. However, the effect is more pronounced in the former. Similarly, lifetime utility increases under all social security and retirement regulation schemes. However, in the case of DA optimal retirement dominates mandatory retirement, whereas the opposite occurs with DC/DB. The reason for this contrasting result is straightforward: with DC/DB, the increase in capital is relatively more important when there is mandatory early retirement. In other words, the gain in dynamic optimality (getting closer to the golden rule) dominates the loss in static efficiency (on the labor market). The opposite holds in the case of DA.

Next, we investigate the transition dynamics between two static equilibria defined by a change in fertility or longevity.

5 Dynamics

Previous sections demonstrated the changes in equilibrium values of capital accumulation and lifetime utilities. In this section, we are particularly interested to see if transition can be welfare worsening for some generations. In other words, we investigate if there are any cases where the short term impact of demographic shift is different than the long term implications.

The simulations in this section use the same parameter values as in the static simulations in the previous section. However, in this case, we need to specify the magnitude of demographic transition between two steady states as a single value. In order to make comparable the changes in n and ℓ , we assume that a fertility driven aging emerges from a decrease in n from 0.2 to 0.137, and a longevity driven aging arises from an increase in ℓ from 0.9 to 0.95. Both changes imply the same magnitude of aging measured by the increase in the rate of dependency $\frac{\ell}{1+n}$ (about 5.6 percent increase from the initial steady state dependency ratio), and both changes are anticipated by the agents. These demographic shocks leads to new equilibria over time, however, not necessarily monotonically as we found.

Figure 2 shows the dynamic adjustment paths in different social security systems and retirement age schemes under different demographic shocks. In the case of a longevity shock, transitions are generally smooth and they have the same sign as the comparative statics with the exception of DA system. In this case, the capital per worker and utility slightly overshoots the final steady-state value for the generation born during the shock. This can be explained by the fact an increase in longevity increases both pension benefits and social security taxes in DA. However, when the shock hits, the transition generation faces only the benefit side of this adjustment as their contributions in young age were done in the initial steady state. Over time, adjustments in second period work and increases in savings bring the system back to a new steady state, where the young generations contribute more to the social security system as compared to the initial steady state.

The most striking result is a transitory loss in lifetime utility that comes with a fertility shock, particularly in DC case. This temporary reduction is due to a loss in old age consumption when the shock hits. Intuitively, a lower fertility reduces the retirement benefits, which leads to a direct decrease in utility for the old. The new generations, however, adjust by increasing their savings in order to finance a portion of this loss in second period consumption. In equilibrium, this increase in savings, together with the reduction in labor force, leads to an increase in real wages. As a result, although the transition gen-

Figure 2: Dynamics of Adjustment under Different Shocks with Alternative Pension Systems and Regulations

Notes:

The parameter values used in the simulations above as follows: In all cases $A = 10$, $\alpha = 0.33$, $\beta = 0.25$, and $\gamma = 0.15$. These reflect commonly used calibrations such as the share of capital in the economy and time preferences. In addition, the following assumptions were made when needed: $a = 1.6$, $\bar{z} = 0.394$ in defined annuity case with mandatory retirement, and $\bar{z} = 0.579$ in defined benefit and defined contribution cases. These values are chosen to make the initial conditions and changes comparable with alternative social security systems. Longevity and fertility rates are kept constant in fertility and longevity shock scenarios, with values $l = 0.9$ and $n = 0.2$, respectively. The longevity shock is an increase in l from 0.9 to 0.95 in period 5, and the fertility shock denotes a decrease in n from 0.2 to 0.137 in the same period.

eration experiences a loss in lifetime utility, the reduction in fertility brings the economy closer to golden rule savings, and lifetime utility increases for the following generations. In comparison, this transitory cost does not appear in the case of an aging driven by increasing longevity.

Overall, the dynamic simulations show that the defined contribution system dominates the other unfunded social security systems in increasing capital and utility as a response to aging in the long term. This can be seen by comparing the changes in equilibrium capital per worker and lifetime utility as a share of the initial equilibrium values displayed in figure 2. However, this long term performance comes at a cost of utility losses in the transition generation.

Finally, note that mandatory early retirement fares better than flexible retirement in its response to aging. Capital accumulation and utility gains are greater with mandatory early retirement with all social security systems; however, these gains are small.

6 Conclusions

In this paper, we evaluate the implications aging on capital accumulation and lifetime utility. We show that these implications differ both quantitatively and qualitatively depending on the type of social security system, type of retirement regulations and the time frame of the analysis. The effects of an increase in longevity or a decrease in fertility, two phenomena that contribute to aging, change depending on these features of the pension system. In a standard overlapping generations model with no social security or second period work, a decrease in fertility and increase in longevity should lead to an increase in capital accumulation. However, this changes when we introduce these elements. The effect of a decrease in fertility on capital accumulation is still positive with a defined contribution system regardless of the retirement age regulation, whereas the results are ambiguous with other unfunded social security systems and with an increase in longevity.

Our findings have important implications, as we know that countries do not age the same way and social security systems tend to shift progressively from a regime of defined benefits towards one of defined contributions. Our dynamic simulations show that this shift could bring long term gains; however, the transition is likely to have welfare costs for current generations. On our further research agenda, we intend to look at the joint effect of aging and changes in the social security regimes explicitly: from DA/DB to DC and from early retirement to flexible retirement. Another limitation of the current analysis is the assumption of identical individuals. With heterogeneity in wages, we could find

more merits in the defined annuity formula, which is also left for future research.

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Appendix

$$G_{1t} \equiv (1+n+z_{t+1})k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{\tau}}{1+\beta\ell} \left(\beta\ell + \frac{(1+n)k_{t+1}^{1-\alpha}}{A\alpha} \right) \quad (27)$$

$$+ \frac{z_{t+1}k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma\ell} \right) = 0 \quad (28)$$

$$G_{2t} \equiv (1+n+z_{t+1})k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{p}}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) \\ + \frac{z_{t+1}k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma\ell} \right) = 0 \quad (29)$$

$$G_{3t} \equiv (1+n+z_{t+1})k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{a}(\ell-\bar{z})}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) \\ + \frac{z_{t+1}k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{z_{t+1}}{2\gamma\ell} \right) = 0 \quad (30)$$

$$G_{4t} \equiv (1+n+A(1-\alpha)k^\alpha\gamma\ell)k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha \\ + \frac{\bar{\tau}}{1+\beta\ell} \left(\beta\ell + \frac{(1+n)k_{t+1}^{1-\alpha}}{A\alpha} \right) + \frac{A^2(1-\alpha)^2\gamma\ell k_{t+1}^{1+\alpha}}{2(1+\beta\ell)A\alpha} = 0 \quad (31)$$

$$G_{5t} \equiv (1+n+A(1-\alpha)k^\alpha\gamma\ell)k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha \\ + \frac{\bar{p}}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) + \frac{A^2(1-\alpha)^2\gamma\ell k_{t+1}^{1+\alpha}}{2(1+\beta\ell)A\alpha} = 0 \quad (32)$$

$$G_{6t} \equiv (1+n+D_t\gamma\ell)k_{t+1} - \frac{\beta\ell}{1+\beta\ell}A(1-\alpha)k_t^\alpha + \frac{\bar{a}(\ell-D_t\gamma\ell)}{1+\beta\ell} \left(\frac{\beta\ell}{(1+n)} + \frac{k_{t+1}^{1-\alpha}}{A\alpha} \right) \\ + \frac{D_t\gamma\ell k_{t+1}^{1-\alpha}}{(1+\beta\ell)A\alpha} \left(A(1-\alpha)k_{t+1}^\alpha - \frac{D_t}{2} \right) = 0$$

Where $D_t = (A(1-\alpha)k_t^\alpha - \bar{a})$