Modeling the Marginal Value of Rainforest Losses

A Dynamic Value Function Approach

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Abstract

A rainforest can be modeled as a dynamic asset subject to various risks, including risk of fire. Any small part of the forest can be in one of two states: either untouched by forest fire, or already damaged by fire, in which case there is both a local forest loss and increased dryness over a broader area. In this paper, two Bellman equations are constructed, one for unharmed forest and a second for already burnt forest. The analysis solves the two equations for the total expected asset values in each of the two states, assuming that asset returns have a constant growth rate over time. The equations are

used for deriving the marginal value of standing (unburnt) rainforest, equivalent to the expected discounted value loss when losing a small additional forest patch. The paper shows that marginal forest value is increased by the additional dryness and forest fire risk that follow from forest fragmentation when additional forest is lost locally. Both forest fires and dryness here serve as “multipliers” to the basic services return loss, within and outside the forest. The paper also presents a framework for calibrating the impact of the forest fire risk component on forest value.

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1. Introduction

The objective of this paper is to build a simple model framework for studying the interactions between forest losses, forest fires, forest dryness, and forest fragmentation. To achieve this, I derive analytically tractable expressions for the “marginal value” of rainforest losses that reflect the main factors contributing to it. Among such contributing factors are the more standard stock and flow value impacts of forest losses, where the impacts themselves can occur both within and outside the forest, and the impacts of forest fires that could potentially be avoided, including how the likelihood of forest fires depends on initial forest losses and the endogenous effects of forest fragmentation. Analyses of such processes are found, for example, in Mendonça et al. (2004) and Soares-Filho et al. (2012). In the latter reference, a model, FISC, developed by a research team at the Federal University of Minas Gerais, Brazil, is presented which forms the modeling framework for fire activity in the Brazilian Amazon. The framework also is applicable to other forest areas.

The analysis provides a basis for considering forest fire avoidance as a separate item in the marginal valuation of rainforests. Such a value item has, to my knowledge, never before been modeled formally. It has however been discussed informally, by Andersen et al. (2002), who evaluated forest fire avoidance (or interactions of forest fires with forest losses) as, in their view, one of the three most important value items associated with forest losses, alongside carbon and sustainable timber value extraction. In this paper, I consider the possibility that such a value component can be quantified and parameterized, principally in the context of the FISC model. A parallel element of the analysis is to consider dryness (or its avoidance), which has both direct impacts on forest value via forest returns both within and outside of the biome, and indirect impacts via changes in forest fire risk, as a similar issue for valuation.

2. The model

The model presented here can apply to both “micro scales” (a particular plot of forest subject to immediate deforestation and its control), and “macro scale” (deforestation occurring throughout the biome, or in relevant major parts of it). To construct the model, we define the following variables:

$I-L$ = currently remaining forest on a unit of initially fully forested land area, so that L is already lost forest. In the following $I-L$ will be taken to have two alternative interpretations: as remaining forest on an

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individual plot; and also as the average remaining forest per plot for the entire forest; which is valid will depend on model interpretation. These interpretations will be equivalent for our purposes under a “linearity” (or proportionality) assumption whereby all impacts of forest losses are proportional to the amount of forest lost, over the relevant range for overall forest losses considered here (up to a maximum of 10% by 2050). Linearity here presumes (plausibly) that no catastrophic developments (such as massive increased dieback or massive extinctions) will occur within this range.

We do not discuss any reasons for past forest loss on a given plot. This could be due to past fires or logging (legal or illegal), or possibly even because there may never have been forest on the plot. We also consider the “snapshot” situation as prevailing today; more forest could be lost in the future but this is not part of today’s picture.

F = forest lost on average from a typical fire event on the unit of land considered, likely to occur in the future. We here for simplicity assume that the magnitude of F is exogenous, and does not depend e.g. on $I-L$ (how much forest is remaining on the plot); we may here assume that L (the fraction of the forest already lost before a future fire event) is relatively small. Any incidence of such fire is limited by $I-L$, and is usually or on average less (some forest remains on a given plot where a fire occurs).

Local fires may in reality take a two- (or even multi-) stage form, where a second stage often having as serious, or more serious, consequences. This is not modeled explicitly in the main presentation, but such a case is presented briefly in an appendix.²

D = forest dryness, which is assumed to be a function of the amount of already lost forest, $D(L)$, where L is now interpreted as aggregate (macro) amount of rainforest lost, with $D'(L) > 0$ (more lost or less remaining aggregate forest makes the remaining forest drier, via various hydrological processes).

Note that dryness will also be affected by the amount of forest lost due to fires. This is not represented formally here, as it leads to few fundamental additional insights at least in relatively simple cases where “tipping points” are not reached, something we assume.³

r = periodic interest rate for discounting of future costs and benefits, assumed constant.

λ = intensity (continuous-time flow probability) of fire occurrence, where $\lambda = \lambda(L, D)$ with $\lambda'_L(L, D) > 0$, $\lambda'_D(L, D) > 0$ (less forest on a given plot, and more generally drier forest, both raise the probability of fire

² Concretely, when a second fire follows relatively soon after the first (say, within a year), we may consider two fire events taking place at a given site as collapsed into one event. When the two are farther apart in time, we consider them as separate fire events.

³ Tipping points may be reached when the interaction between dryness and fires takes a more catastrophic form. Such possibilities will not be pursued here.

on the plot within a given period of time). The occurrence of forest fire on a given plot of unit size is here considered as governed by an associated stochastic process with constant transition probability (or intensity) λ . We assume that forest fires are governed by random processes. In reality fires occurring in forests such as the Amazon often are set by humans (Mendonça et al (2004)), but this is not considered explicitly here. We also assume that fire can occur only once on a given plot. The appendix develops an example to two possible fire occurrences on a given plot, often considered more realistic. The appendix however shows that the two-fire case can readily be collapsed to the case of only one fire event, with a simple reinterpretation of the parameters F and λ .

θ = intensity of occurrence (in terms of flow probability) of exogenous forest loss on a given unit plot, whereby all remaining forest on that plot is lost. The most obvious factor behind such events is “dieback”, a catastrophic (local, regional or biome-wide) transition of forest to non-forest land for reasons not discussed nor modeled in detail here, but could be assumed to be governed by separate and exogenous stochastic processes assumed here to be independent of other variables. In the literature “dieback” is often considered to depend on various other factors among those some discussed here, including dryness and average forest fragmentation; see e.g. Nepstad et al (2008), Vergara and Scholz (2011). Another possible interpretation is illegal clear-cutting by logging, which we also take as exogenous and independent of factors discussed here.⁴

In interpreting results from the model, we may alternatively interpret θ as background or baseline forest loss more generally.

Fire occurrence is also assumed to follow a Poisson process with constant parameter λ (which however depends on L and D as stated above). A property of these processes is that expected time to the occurrence of, respectively, a forest fire, and dieback, are $1/\lambda$ and $1/\theta$.

v = per-unit (current or flow) value of services and goods provided by remaining forest. v can be interpreted either as a local or (biome-wise) average per-period return. Which interpretation is valid will depend on required interpretation of total versus marginal forest values; and the way in which forest is lost; see below. Generally, v (interpreted as a returns from a plot) could be highly differentiated across the biome.

On the other hand, dryness D is considered in this presentation as a biome-wide phenomenon. It is assumed to affect standing forest values negatively through the function $v(D)$, which now must be interpreted as a single unified function for the entire biome. The derivative $v'(D)$ (< 0) then takes a single

⁴ This interpretation is more controversial as it relies on values extracted as a result of illegal logging as not being part of a social surplus as discussed here.

biome-wide value only. Rather generally, greater forest dryness has a negative effect on forest value. This can occur through reduced returns in terms of forest products and services, and/or reductions in biodiversity and biological resources.

w = per-time unit returns to a given unit of rainforest, via services and goods outside of the forest itself. This can be an especially important factor in South America, in particular for the impacts of rainforest-generated rainfall on hydropower and agricultural production, which for the most part occurs outside of the Amazon biome. We assume a negative relationship between this value and average dryness D , so that $w'(D) < 0$ (a drier forest transports less moisture to the surrounding region thus reducing yields), and this effect must be analyzed on a macro scale for forest changes.

g = (exogenous) rate of growth of forest returns over time, applying to all current terms entering into the forest value. We assume (as a basic stability condition) that $r-g > 0$. It is easiest to think of a positive g as representing a combination of physical forest growth, increased scarcity (and/or prices) of harvestable resources (such as timber), and increased willingness to pay, for example, for biodiversity protection. If the forest were instead deteriorating exogenously over time, g could in principle take negative values (although increases in economic valuation of the forest over time would more typically lead to $g > 0$).

We use a highly simplified value function and dynamic programming approach to derive the relevant asset equations for two current states: an initial state of the relevant forest plot (before the occurrence of fire); and the state after fire has occurred. This yields very simple solutions when the transition probabilities λ and θ are constant, as assumed here; and when (as also assumed) fire on a given plot can occur only once.⁵ Define the current asset value of a forest plot before a fire event occurs, V , as given by the (continuous-time) asset equation (also called Bellman equation; Bellman and Kalaba (1965), Bellman (2010); for relevant applications to other areas see e.g. Stokey et al (1989)); Strand (2006)):

$$(1) \quad rV = (1 - L)[v(D) + w(D)] + l(L, D)(V_F - V) + (g - q)V.$$

Equation (1) embeds the terms that produce the aggregate returns, rV , to the capitalized asset value of a given small part of the rainforest, V (focusing on a given forest plot or pixel, assumed to currently have an amount $1-L$ of forest). The most direct such returns are the two main value streams per time unit (flow values), $v(D)$ (current flow value of benefits from the forest itself, per unit of forest), and $w(D)$ (current flow value of benefits produced per unit of forest, within the region but outside of the forest). Both these value components are assumed to be functions of general forest dryness. In addition there are three terms expressing (probabilistic) value changes. The first (which is negative) represents loss of value due to fire

⁵ As already noted, the case where fire can occur twice is considered in an extension to the model, in the appendix.

risk; this is thus the “fire-induced” loss; the second, gV (positive), represents value growth over time; and the third, $-\theta V$ (negative), represents the risk of dieback.

Note that equation (1) has no direct representation of two important value categories related to tropical forest and its loss, namely timber values, and carbon values. Equation (1) can be viewed as valuing the forest in a steady-state with no forest extraction for timber purposes, or alternatively with timber extracted sustainably (so as not to reduce the timber stock and not harm other forest services). When deliberate deforestation occurs, and the timber on the respective plot extracted, we will add a net value term T per unit of L that is lost, to represent timber values; see equation (6) below.

Carbon values are as noted not represented directly, but are here generally viewed as rental values and belonging alternatively to v or w . It could be viewed as a value intrinsic to the forest as carbon is retained (when forest is saved) or released (when forest is lost) directly by the forest itself. It could however also be viewed as part of the term w , as a carbon value factor belonging to the rest of the world beyond the biome, and with a flow value representing the avoidance of its loss. For the moment we do not specify the way in which such values enter this term.⁶ Note that we do not with this formulation explicitly represent any possible effects of additional carbon being captured by existing forest.

V_F given by (2), represents the forest value for this plot after a fire has occurred, given in turn by a similar Bellman (or asset value) equation:⁷

$$(2) \quad rV_F = (1 - L - F)[v(D) + w(D)] + (g - \theta)V_F.$$

The elements of (2) have similar interpretation as those in (1).

The two equations (1) and (2) can be solved recursively, as follows (where (2) solves for V_F only):

$$(3) \quad V = \frac{1}{r - g + \theta} \left[1 - L - \frac{\lambda}{r - g + \lambda + \theta} F \right] [v(D) + w(D)]$$

$$(4) \quad V_F = \frac{1}{r - g + \theta} [(1 - L - F)[v(D) + w(D)]].$$

⁶ An option not pursued here would be to view only values to be captured by carbon markets or REDD+ schemes as belonging to the region via either v or w .

⁷ One here needs to recall our assumption that no further fires occur after a first fire, that a burned forest does not recover.

We immediately see, from (3), that increasing expected damage due to future forest fires reduces the value of the standing forest on a given plot. This loss is strongly increasing in both λ (the fire risk) and F (the average loss of forest given that a fire occurs).

The formulation in (2) here reflects our assumption that only one fire event will ever occur on any given plot (which can be taken as an approximation given that λ (the forest fire risk) is relatively small, and/or a given fire event consumes much of the remaining forest on this plot). Remember also that λ is a function of L and D .

Another key assumption is that forest fires do damage to the forest only. In reality forest fires often also damage crops when a forest fire occurs close to cropland (which is likely to often be the case for example in the southern and eastern parts of the Brazilian Amazon); and also potentially damage to human health; see Mendonça et al (2004). In terms of accounting for overall forest fire damages, such additional effects can however in a rather simple way be embedded in the analysis by adding them to other costs.

In interpreting (3), care must be taken not to mix such or similar plot values with total values of a larger forest biome. A common conclusion, when discussing total versus marginal global resource values (referring in particular to analyses of the type done by Costanza et al (1997); see e.g. Toman (1998)), is that as ecosystems are lost, the marginal value of the remaining system is likely to increase. This is not represented here (nor in the analysis below where we discuss marginal changes both at the local and macro biome levels). Importantly, we here focus on particular (marginal or small) parts of the rainforest; and our overall perspective is in any case, in our valuation context, to value only marginal changes. We do not find it meaningful to consider total rainforest value for a large biome such as the Amazon.

3. Marginal value of rainforests

The key issue in the context of a geographically differentiated rainforest value is the additional or marginal losses of economic value that result from a marginal or small (exogenous) forest loss occurring either in specific parts of the biome, or more generally throughout the biome. Alternatively, we seek to derive incremental forest value gains due to more rainforest being preserved, or to reductions in rainforest losses. One objective of such analysis is to study how the forest fire component affects these marginal values: how large is the increased loss of forest, and/or forest value, via the forest fire component, when a (small) amount of forest is lost exogenously (to dieback or other exogenous mechanisms)? The question can also be put positively: what is the additional forest and/or forest value saved through reduced forest fire frequency, when an additional small forest plot is protected from deforestation today?

We will also use the model to study other similar impacts, including those related to changes in forest dryness. We then visualize the marginal (exogenous) changes in forest cover to occur at either the micro (the individual small plot) or the macro scale (simultaneously in all micro plots making up the macro biome). Recall that dryness is a macro phenomenon applying to the entire biome; while forest fire risk is affected locally.

Such marginal forest value analyses can be conducted separately for forest areas that are initially intact (undamaged by forest fire), and areas that are at the outset damaged. We focus in this presentation only on the former category.

We see from the analysis above that the effect via the forest fire component here occurs only through the impact of initial forest losses on forest fire risk, λ . We assume for simplicity that the extent of the loss of forest in a plot due to any given forest fire, F , is exogenous and constant, and is thus by assumption not affected by the magnitude of initial forest losses (locally or more widely in the biome), nor by other variables included in this model. A measure of the gain resulting from an exogenous (additional and small) gain in the size of the forest, $1-L$ (or equivalently, the additional loss associated with a small increase in L), is obtained by taking the derivative of V in (3) with respect to L .

The discounted social gain from avoiding one unit of deforestation, on plots which have not yet been subject to forest fire, takes the two alternative forms, distinguishing between two types of forest loss, “spontaneous” and “deliberate”:

$$(5) \quad -\frac{dV}{dL} = \frac{1}{r-g+\theta} \left[1 + \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \lambda_L F + \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \lambda_D D' F \right] (v+w) - [v'(D) + w'(D)] D' \frac{V}{v+w}$$

$$(6) \quad -\frac{dV}{dL} = \frac{1}{r-g+\theta} \left[1 + \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \lambda_L F + \frac{r-g+\theta}{(r-g+\lambda+\theta)^2} \lambda_D D' F \right] (v+w) - [v'(D) + w'(D)] D' \frac{V}{v+w} - T$$

Equations (5) and (6) represent net marginal forest values given two main alternative ways of losing rainforest. In (5), forest is assumed to be lost spontaneously due to exogenous factors, such as through dieback or illegal logging, where it is assumed that no resource values are realized as a result of forest being lost (so that when, in particular, forest is lost due to illegal logging, no net value to illegal loggers is considered as part of a social surplus).

Equation (6) represents the marginal net value of forest preservation under the main alternative mode of forest loss, clear-cutting through logging, where a net timber value T per unit of forest area would be

realized upon deforestation. In either case we assume, for analytical simplicity, that lost forest is lost forever.⁸

In equations (5) and (6), the terms containing F represent impacts of forest loss on overall forest value due to the increased fire risk that occurs when more forest is lost due to increased forest fragmentation and dryness. The last terms, which include D' , represent impacts of forest losses on values (both within and outside of the forest) which follow from increased general forest dryness, within and outside the biome.

To consider factors that influence on the marginal forest value as found from (5) (or (6)). We see that the effective discount rate $r-g+\theta$ is central for this value. When interpreting θ as an exogenous “baseline deforestation rate”, one finds that a higher such a rate makes forest values lower, both in total and at the margin. The intuitive reason is that saving forests today is less valuable when we know that the forest will, in any case, disappear relatively soon (as given by the “baseline” forest reduction rate). This may appear as a “defeatist” approach, in particular if high baseline deforestation leads to low protection values which in turn might lead to saving less of the forest. Note however that the latter is a less likely outcome as higher baseline deforestation implies also similarly lower opportunity values related to saving the forest now.

A difference for the model, when considering local versus general biome-wide forest changes, is related to the interpretation of the forest-specific value v . This variable takes a biome-wide value in general (not plot-specific) forest loss cases; and in the biome-wide change case. Our linearity assumption (whereby overall valuation impacts of forest losses are proportional to amount of forest lost) here leads to a substantial simplification as, when analyzing forest changes, and given that site-specific values are all the same, it does not matter whether a marginal forest change occurs on a given plot, spread across many plots, or on the average plot, the marginal valuation results apply more generally. Also, for dryness we assume that the marginal value $v'(D)$ is invariable independent of any possible plot-specific value of v .

On the other hand, v may vary across plots; to identify its variation across the biome is indeed one of the key elements of our project. In cases of plot-specific forest losses this is manifested only through v as it enters into the first major term in the right-hand sides of (5) and (6), which can be highly variable. We however assume that w entering into the same terms take a biome-wide value.

⁸ We ignore the fact that much commercial logging in forests such as the Amazon is selective and does not lead to land clearing. The argument here is imprecise also because when forest is lost to dieback there will often be a remaining net timber value which can be realized. Dieback may however occur in inaccessible areas where the cost of bringing timber to market is high.

The relationship (5) also captures the impacts of endogenous dryness within the forest biome and how it depends on the amount of forest remaining in the biome. For any one particular (small) plot of forest, overall dryness might (possibly) be considered exogenous, as it is affected (only or mainly) by macro variables. But this is a faulty line of reasoning when considering impacts of particular forest losses on the forest as a whole. The reason is that when forest disappears on a plot, this has externality impacts on all other plots. While the effect on each plot could be negligible, the aggregate effect on all plots is of the order of the effect on $I-L$, now interpreted as the average amount of forest on all plots (or the share of remaining forest in the entire biome). We are reasonably assuming that $dD/dL = D' > 0$ (so that when more forest is lost, the drier will the remaining forest be on average). The same reasoning would apply under a linearity assumption, invoked throughout this paper, whereby overall impacts on the biome are proportional to the amount of forest lost or gained regardless of where the loss occurs. This relationship can consequently be used in an extended analysis of forest losses. We see that the second term in the square bracket in (5) stems from the forest fire risk change. We will, correspondingly in the following, consider a relation corresponding to (5) for the case of effects via changed rainforest dryness, the third term in the square brackets.

The first of the dryness effects, the last term inside the square bracket of (5), represents the macro effects of increased dryness on forest fire incidence when additional forest is lost throughout the macro biome. This is the only additional forest-fire related impact here.

The second term related to dryness, which follows after the square bracket, represents the direct value loss as the standing value of all forest is negatively affected by increased dryness, via the function $v'(D)$. $V/(v+w)$ can here be interpreted as a (value-related) measure of remaining forest size. This impact is related to other issues and factors than forest fires, which are not specified here. They could relate, most obviously, to hydrological impacts, for example, on agriculture and hydropower production, but could also be related to changes in biodiversity as the forest dries, reduced timber growth, or reduced potential for harvesting non-timber forest products. The marginal effects, both the effect of D on v , and that of L on D , are likely to be very small for any given plot. But note that these effects apply to the entire standing forest making for a potentially significant aggregate effect (and this is what is here meant to be represented by the functions $v(D)$, and $D(L)$).

For this model to provide geographically differentiated forest values across the biome), (5) and (6) should express marginal values of forest on individual small plots of forest. Analysis of expressions (5) and (6) can then serve to inform optimal forest management policy decisions, where the policy alternatives are to preserve the forest, or clear-cut it with the objective to extract all the standing timber. With this

interpretation, T must embed also the value of the deforested land in alternative uses (such as agriculture).⁹

Note that as long as T is sufficiently small so that $-dV/dL$ remains positive in (6), deliberate deforestation with timber harvesting is not an optimal policy. T should then not be included in the expression for forest value, V , in (3). This will be the standing hypothesis here and in the following. Equation (6) will then not play any further role for our discussion.

The expression (5) then provides a first approach to the analysis of total marginal value of saving one additional unit of forest, instead of losing this unit of forest now (due to other reasons than forest fires or deliberate logging). It also gives us a hand on the following issue: what is the share of value gain that follows from the impact through reduced frequency of forest fires? It is here natural to associate the first term in the square bracket in (5) (apart from the 1) with non-fire effects, and the second term with fire effects.

Equation (5), as already mentioned, also captures the externality cost of forest loss on the world outside of the forest itself, here represented by $w > 0$, which increases the marginal forest value. Less forest in the biome leads to less moisture transported out of the biome, with resulting negative effects on the outside world. This is a standard externality costs associated with deforestation. It is specifically relevant for our project in terms of hydrological changes which have impacts on agriculture and hydroelectric production outside of the forest area.

We here find, first, that the marginal forest value (equal to the marginal gain from retaining an additional unit of forest; or avoid a unit of deforestation) is *greater* than the average per unit forest value, found from (3). The reason is the externality cost through greater fire risk resulting on a particular piece of forest, and a similar contribution to greater general forest dryness across the biome, when the remaining forest on this piece becomes more fragmented.

Note that the resulting fire risk probability is at the margin reduced by the increase in $I-L$ (or decrease in L). (Note then also again, importantly, that we assume constant marginal forest value v for any individual plot and in aggregate; thus as noted, no effect on marginal forest value from the resource becoming scarcer as the amount of forest on the plot drops.)

It is finally worth mentioning that the carbon release effect of forest loss could depend greatly on the form that deforestation takes; this is not taken into consideration in deriving (5) and (6). First, when timber is

⁹ Alternatively, it may not be economically optimal to clear-cut a given plot. In that case, T will represent simply the net value of the extracted timber, with no alternative-value component.

accumulated as usable wood products (including as building materials and furniture), there is likely to be no or only a small immediate carbon release upon deforestation; and the subsequent carbon release (as when building materials or furniture rot) may take a long time (up to several hundred years). Additionally, as shown by Oliver et al (2014), using timber products as building material implies an additional net global carbon emissions mitigating effect by reducing the need for other, more carbon-intensive, building materials such as steel and cement. Thus from a global perspective, there may be reason to include such positive effects in the impacts of logging (in (6)), when the alternative is forest loss with full carbon release, as we assume as the basis for (5). This might in some cases tip the balance in favor of clear-cutting by logging, by making $-dv/dL$ negative in (6).

We can sum up the discussion in this section by identifying (at least) seven separate effects of primary forest losses, that add to net economic value losses (or net economic value of preserving forest) at the margin:

- 1) The direct and primary loss of forest services when a forest plot is lost. This primary value loss is likely to vary substantially across the biome, and is the principal target of our valuation effort.
- 2) Additional losses via impacts on the frequency of forest fires, through forest fragmentation. In our model, fire frequency is common for the entire biome; with more detailed data this impact could be thought of as differentiated.
- 3) Losses as increased macro (biome-wide) dryness leads to more frequent forest fires. This is a biome-wide effect.
- 4) Losses in the average flow return per unit of standing forest from greater dryness within the macro biome.
- 5) Additional losses due to externality impacts on economic activities outside of the forest biome, when forest is lost in the biome, mainly also through greater dryness.
- 6) Timber values are positive (and alternative to other forest rents) in the case of clear-cutting by logging (as timber is then extracted for sale), while they are part of the flow return to the forest when kept intact.¹⁰
- 7) The (global) carbon release impact of alternative modes of forest losses, with focus on differences between spontaneous forest loss, and logging.

The primary forest loss, fragmentation impacts, and timber value (1-2 and 6) can here be considered to apply at the micro level (on the each individual plot only), while impacts 3-5 can be considered to work

¹⁰ We here ignore any possible pecuniary externalities whereby increased timber extraction may reduce the timber price, as studied in Harstad and Mideksa (2015).

via the macro biome impacts. Impact 7 is a purely global effect. This perspective can be of consequence when the model is to be parameterized based on empirical data.

A possible implication of this analysis is that some hydrological and other value components may need to be counted twice in a marginal valuation exercise: First, in terms of their primary value impact; and secondly, in terms of the marginal (incremental) impact on unit values for damages related to other value components. This might appear as double counting but it is not, as the two cost items accounted for represent direct and externality cost effects, both of which ought to be counted.

Finally (focusing on initially undamaged forest), note that while *fire risk reduces the average value of a given stock of standing forest* (through the expectation of future such losses; in present value terms, from (3)), *fire risk at the same time increases the marginal forest value*, expressed as the value of saving one unit of forested land from being deforested (as this risk is reduced by preserving more of the current undamaged forest; from (5)). It is principally the latter, *geographically differentiated concept that we seek to derive here*.

Note also the result that *higher baseline risk of forest losses reduces the present value of saving forest today, both in total and at the margin*. But on the other hand, also opportunity values are similarly affected, so that decisions to save or not save the forest are not necessarily (seriously) affected.

4. Calibration and estimation of the derived marginal value relationship

A realistic calibration of the various components and contributions to marginal forest value in the model, and identification of their factual importance, should depart from (5), which expresses the marginal value of retaining one unit of forest (when disregarding any possible one-time timber extraction value associated with the forest loss through clear-cutting). Focus is on the relative contributions of the forest fire and hydrological components to marginal forest valuation.

For a preliminary assessment of parameters that will need to be identified for such a relationship to be calibrated and estimated, define the following key parameters:

$$(7) \quad El\lambda(L) = \lambda_L \frac{1-L}{\lambda}$$

$$(8) \quad El\lambda(D) = \lambda_D \frac{D}{\lambda}.$$

Here, $El\lambda(L)$ and $El\lambda(D)$ represent (absolute-valued) elasticities of plot-specific forest fire risk, λ , with respect to $I-L$ and D .¹¹ A relevant observable measure of dryness, D , must then also be chosen; e. g. average annual precipitation. Intuitively, $El\lambda(L)$ expresses the percentage change in forest fire risk, in response to a one percent reduction in the forested area on a given plot and in aggregate for the forest. Correspondingly, $El\lambda(D)$ expresses the percentage increase in forest fire risk, when dryness increases by one percent.

These two parameters would appear to be feasible to assess from the forest fire model in use. The same should λ , the typical fire risk; and F , the typically forest loss given a fire.

To apply the framework, especially with geographically dispersed data, it is important to identify and if possible estimate these parameters, plus the following additional necessary parameters:

λ = the overall average forest fire risk

$El_L D = D'(I-L)/D$ = percentage change in dryness in response to forest area being reduced by one percent

$El_D v = v'(D)D/v$ = percentage increase in overall (forest-internal) rent value from standing rainforest in response to a one-percent increase in forest dryness (which is likely negative).

$El_D w = w'(D)D/w$ = percentage increase in overall rent value outside of the forest biome, in response to a one-percent increase in forest dryness (also negative).

$F/(I-L)$ using geographically dispersed data.

The parameters r (rate of discounting), g (forest value growth), and θ (annual risk of dieback or other exogenous forest loss) also need to be assessed. With information about all these and the above parameters, we can identify the relative contribution of forest fires to value (in total expressed by the square bracket in (5)). One would here clearly also need data for the average fraction of remaining forest, $I-L$, on the average such plot. Finally, a complete parameterization of the model would require that we also know v and w (and V ; the latter value can be derived from other parameters).

¹¹ More precisely, in fact, $El\lambda(L)$ is here defined as minus the elasticity with respect to $I-L$.

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Appendix: Three-state model with two possible fire states

One argument with the model above could be that it treats the forest fire issue in an overly simplistic way, as it is assumed that each plot can burn only once. In the following we present an extended model where each plot can burn twice, which could be a more realistic description.

We denote by F_1 and F_2 the expected amounts of forest that burns at each of the two steps (a first and a second fire on a given plot). Once a fire of expected size F_1 has taken place on a given plot, the plot can still in the (relatively near) future be subject to an additional fire of expected size F_2 , where the total area constraint $F_1 + F_2 < L$ must be obeyed.

We then need three asset equations instead of just two as in the model above. (1) is still valid except that F needs to be replaced by F_1 , where λ would describe the transition rate also in this case.

Equation (2) must be reformulated, as follows:

$$(A1) \quad rV_{F1} = (1 - L - F_1)[v(D) + w(D)] + (g - \theta)V_{F1} + \mu(V_{F2} - V_{F1}).$$

where μ is the continuous rate at which a second forest loss occurs on the plot in question, and V_{F2} is the corresponding value function after such a loss. We now have a third equation determining the latter value function, given as

$$(A2) \quad rV_{F2} = (1 - L - F_1 - F_2)[v(D) + w(D)] + (g - \theta)V_{F2}.$$

We can now solve (2), (A1) and (A2) for V by inserting recursively for V_{F1} and V_{F2} :

$$(A3) \quad V = \frac{1}{r - g + \theta} \left\{ 1 - L - \frac{\lambda}{r - g + \theta + \lambda} \left[F_1 + \frac{\mu}{r - g + \theta + \mu} F_2 \right] \right\} (v(D) + w(D)).$$

We see that V in (A3) is modified relative to (3), by the last term inside the square bracket, which expresses the negative impact of the second fire loss that can occur on given plots. This could comprise a substantial modification if F_2 is large relative to F_1 , and μ is large (so that there is a high probability of a second fire on one plot, given that one fire has already occurred there). On the other hand one might easily think of adjusting the term F in (3) so as to match the entire square bracket in (A3). Thus, we will argue, the presentation in the text above is sufficiently to usefully describe also the more complicated case of two possible plot fires, instead of just one.